FRACTAL INSPIRED MODELS OF NUCLEON STRUCTURE FUNCTIONS AT SMALL BJORKEN - X

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ABSTRACT

The paper reports an application of fractal geometry in physics of strong interactions. In collaboration with R Gogoi ,we have studied the structure of the nucleon in high momentum squared Q^2 and small Bjorken –x using the notion of fractals and self similarity. Specifically , we propose a fractal inspired model for the structure functions of the nucleon , which is an improvement over that of Lastovicka (2002).

The formalism is then applied to deep inelastic electron-proton as well as neutrino –nucleon scattering. We also make predictions for ultra high energy neutrino –nucleon scattering cross-sections. Our predictions are compatible with

Standard QCD based results and are well within the experimental upper bounds estimated from observed neutrino flux. Fractal inspired gluon distribution inside the nucleon is also proposed and its possible application at LHC is discussed.

PLAN OF THE TALK

- Introduction to self similarity, fractals and fractal dimensions
- Some Fractals in classical and quantum mechanics
- Application to QCD
- Self Similar quark densities and Structure Functions: HERA,CCFR,UHEν,LHC

- What is Fractal dimension ?

A fractal is by definition a set for which the topological dimension differs from the fractal dimension and which exhibits self-similar properties. A structure is said to be self-similar if it can be broken down into arbitrary small pieces, each of which is a small replica of the entire structure. The small pieces can in fact be obtained from the entire structure by a similarity transformation. Self-similarity is the property of all fractals. It can be exact, quasi or statistical. Mathematical fractals exhibits exact self-similarity across all spatial or temporal scales, such that successive magnifications reveal an identical structure. Examples are Koch curve and Sierpinski carpet.

SELF-SIMIALRITY AND FRACTAL DIMENSION :

As the length scale of smaller self-similar sub-objects decrease, their number increase. Let N(R) be the number of tiny self-similar objects when the sub-length is R. As R is reduced, R(N) will increase. Let the increase follow a power law:

$$N(R) \propto 1/R^D,$$

$$\text{Or } N(R) = C/R^D.$$

Taking log,

$$\text{Log } N(R) = \text{Log } (C) - D \text{ Log}(R)$$

It yields

$$D = \frac{\log C}{\log R(N)} - \frac{\log N(R)}{\log R(N)}$$

As R(N) $\rightarrow 0$ for sufficiently large N, the first term of (2) can be neglected, leading to

$$D = \underset{R(N)\rightarrow 0}{Lt} \frac{\log N(R)}{\log 1/R(N)}$$

“D” is the exponent of self-similarity, called *Fractal Dimension.*

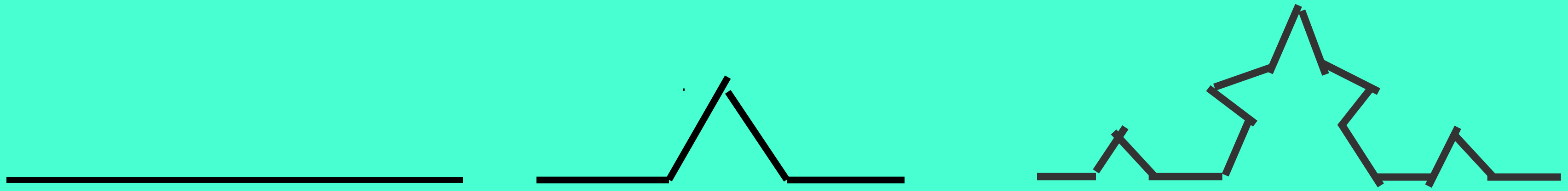
1/R(N) can also be defined as magnification factor M, so that if magnified M times, R(N) recovers the full self-similar object under consideration. As a result, eq (3) can be written in an alternative manner:

$$D = \log M^D / \log M$$

KOCH CURVE

This classical fractal was originally described by Von Koch in 1904.

We start with a line segment of unit length. Now divide it into three equal parts and remove the middle one of length 1/3. In its place, we install two segments of length 1/3 so that both of them meet at the vertex A. We iterate this procedure with each segment ad infinitum.



Koch curve

n=0 n=1 n=2

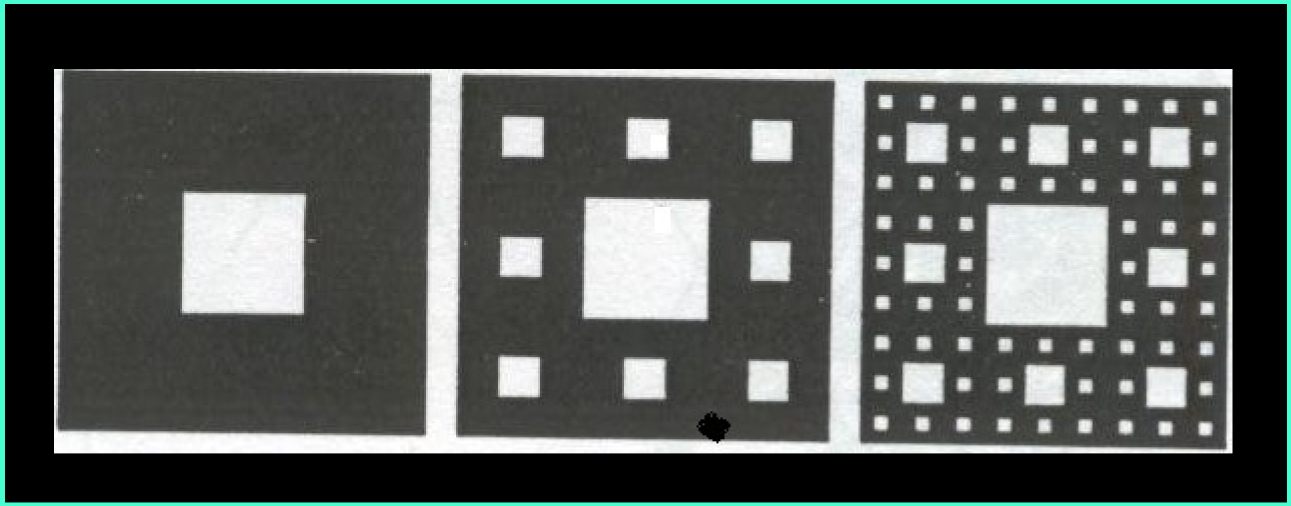
N=1 N=4 $N=4^2$

NR = 11 R=1/3 $R=1/3^2$

$$D=\frac{\log 4^n}{\log\left(\frac{1}{\left(\frac{1}{3}\right)^n}\right)}=\frac{\log 4}{\log 3}=1.26$$

SIERPINSKI CARPET

In this case, one starts with a square carpet of side of length unity. Divide it into nine equal parts and remove the central part. We will be left with 8 squares each of side of length 1/3. Apply the same transformation to the remaining eight 8 squares. One obtains $8^2=64$ squares of sides of length $(1/3)^2$. Iterating the process indefinitely yields the fractal carpet named after Sierpinski.



Sierpinski carpet

n=0	n=1	n=2
N=1	N=8	N=8^2
R=1	R=1/3	R=1/3^2

$$D = \frac{\log 8^n}{\log\left(1 \big/ \left(1/3\right)^n \right)} = \frac{\log 8}{\log 3} = 1.89$$

GENERALISATION OF THE DEFINITION OF FRACTAL DIMENSION

- The definition of fractal dimension may be generalized to the case of non-discrete fractals. In such cases, the number will be replaced by some density function f(M) ,which will be related to the magnification factor M by a power law:

  $$f(M) \propto M^D.$$

  The logarithm of the density function can be written as

$$\text{Log } f(M) = D \log M + D_0.$$

In a more complicated case, fractals may have two different and independent magnification factors M_1 and M_2 for which instead of a single power law, the function $f(M_1, M_2)$ satisfies two power law behavior :

$$f(M_1, M_2) = \text{constant}.\ M_1]_{\text{FIXED } M2}$$

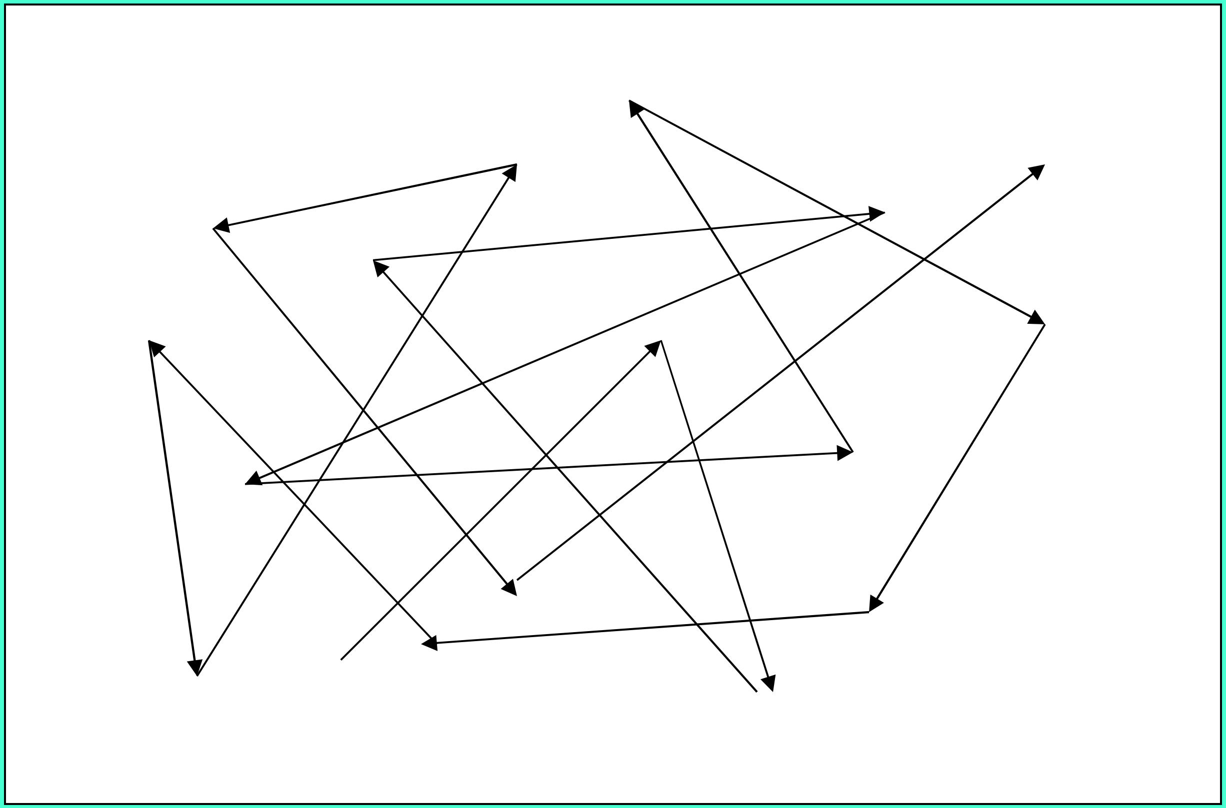
$$f(M_1, M_2) = \text{constant}.M_2]_{\text{FIXED } M1}$$

The logarithmic of density function then takes the form

$$\log f(M_1, M_2) = D_{M1,M2} \cdot \log M_1 \cdot \log M_2 + D_{M1} \cdot \log M_1 + D_{M2} \cdot \log M_2 + D_0$$

where D_{M1M2} represents the dimensional correlation relating the M_1 and M_2 factors. The function $f(M_1, M_2)$ satisfies a power law behaviour in M_1 for fixed M_2 and in M_2 for fixed M_1.

Brownian Motion and Fractal Dimension :



It has been shown by Kroger(2000) that the average path of the Brownian motion is a fractal curve with fractal dimension

$$D = d_{fract} = 2.$$

Plausible Arguments:

If one considers a length L, then the

Length should increase as a power of its smaller segments ε :

$$L(\varepsilon) \approx L_0\, \varepsilon^{-a}$$

With $a = d_{fract} - d_{topo}$.

For Brownian path,

$$d_{topo}=1$$

It has been shown that the main contribution Comes from the configuration

$$(\Delta x)^2 \sim \Delta t$$

Where Δx means the increment of length of an average curve for a given time increment Δt.

Let a total time interval $T=n\Delta t$ is given.

Then, the length of an average curve:

$$L = n\Delta x = (T/\Delta t).\Delta x = T(\Delta x)^{-1}$$

Thus,

$$a = 1 = d_{fract} - 1.$$

$$\Rightarrow D = d_{fract} = 2.$$

Quantum Mechanics and fractals :

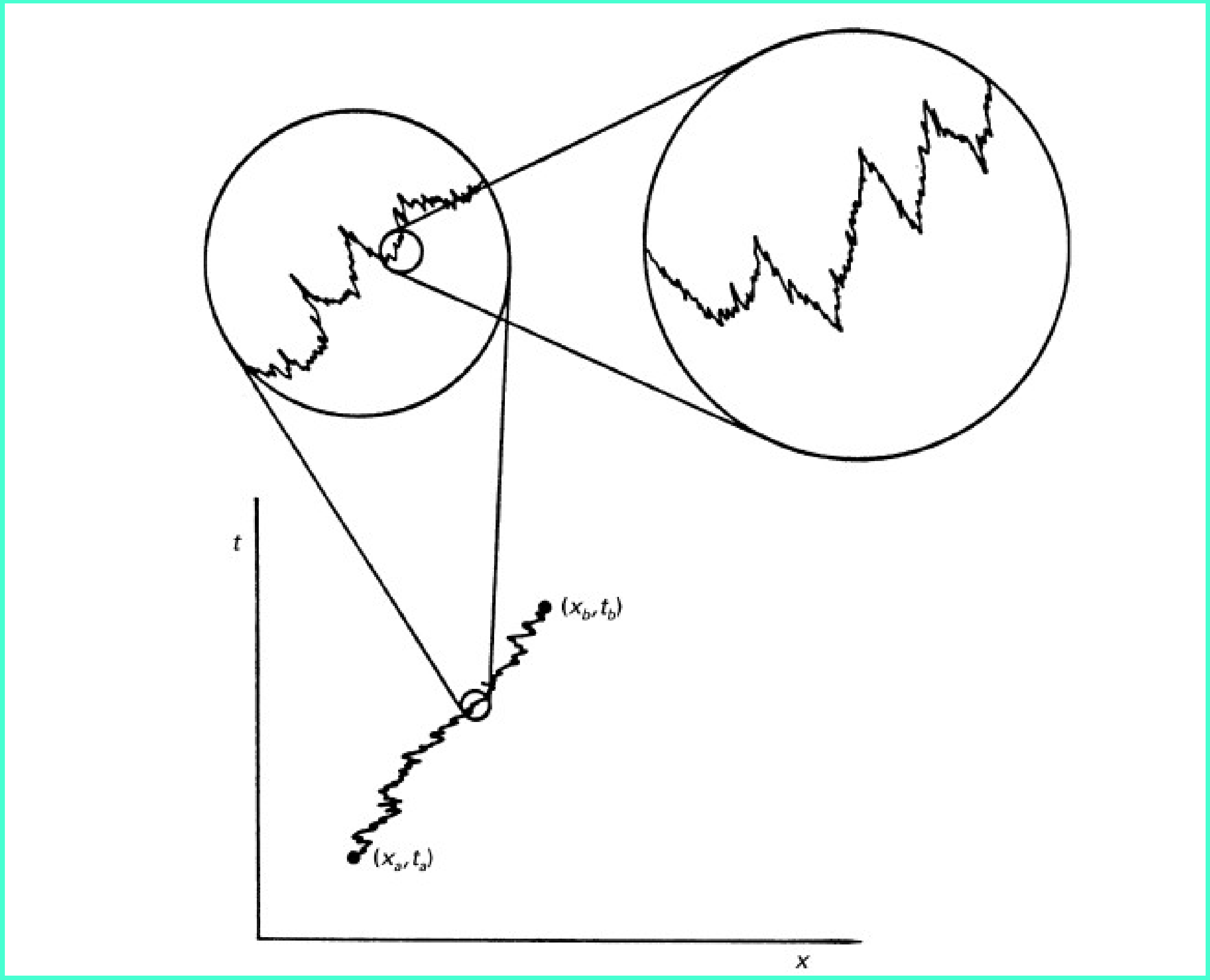


Fig : Typical path of a quantum mechanical particle are highly irregular on a fine scale.

The concept of fractal geometry in Quantum Physics was invoked by Feynman describing that the paths occurring in the path integral have property of fractals. In Feynman and Hibbs, typical trajectories of a quantum mechanical particle are shown to be highly irregular on a fine scale as shown in Figure. In a sense, Feynman found this ‘fractal nature’ of quantum paths though he did not use the term ‘fractal’.

FRACTAL DIMENSION OF A NON-RELATIVISTIC QUANTUM PATH

- Abbott and Wise(1981)
- Particle trajectory of topological dimension 1 in Non –relativistic Quantum Mechanics is characterized by a fractal dimension 2,when the resolution becomes smaller than its de-Broglie wave length $\lambda = \hbar / p$.
- A heuristic argument goes like this:
- Uncertainty relation: $\Delta x \Delta p \sim \hbar$

.

Putting $\Delta p = m.(\Delta x/\Delta t)$

One obtains $(\Delta x)^2 \sim \Delta t$.

When consider the propagation between two given points in space and time, where the time interval T is broken into subintervals of length Δt, the length of the quantum path=>

.

$$\Delta L = N\Delta x = (T/\Delta t).\Delta x \sim T/\Delta x \sim \Delta x^{-1}$$

i.e.,1= D-1

=>FRACTAL DIMENSION OF A QUANTUM PATH;

D=2!

Close relationship between

Brownian Motion and quantum mechanics attributed to correspondence between

Diffusion Equation and Schrodinger Equation(Nelson1966,Kroger2000).

Brownian motion goes over into free motion of a massive quantum mechanical particle with replacement

Time t->it and diffusion coeff d->ħ/2m.

FRACTALS AND QCD

- In relativistic Quantum Mechanics and in particular, Quantum Field Theory,the situation is different from non relativistic quantum mechanics in many ways , because of which application of fractals becomes complicated.
- Time is treated on equal footing as space. Hence particles can propagate

- Forward and backward (antiparticle) in time
- In field theory , the total no. of particles is not conserved and depends on the resolution power of the probe. In QFT, particles can be created and annihilated.
- The position of a particle not observable.

Hence the concept of a particle path is not adequate to evaluate fractal dimension .

One must look at space and time dependence corresponding to a casual propagation of a massive particle.

The situation becomes even more complicated in case of nonabelian field theory like QCD.

ANOMALOUS DIMENSION

In QCD, one encounters 'Anomalous dimension γ,which controls the moments of structure functions.

It is the difference in the dimension of an observable between classical physics and quantum field theory .But it may be solely due to particle creation and annihilation of relativistic QFT and not due to

The geometry of propagator .Indeed in non-relativistic Quantum Mechanics, there is no particle creation and the anomalous dimension $\gamma=0$.So, it does not play the role

of fractal dimension.

Besides, anomalous dimension describes the ultraviolet behavior of QCD, while fractal dimension, infrared physics.

Hence Anomalous Dimension may not throw new light on fractal dimension.

How to bring this notion to QCD at least at

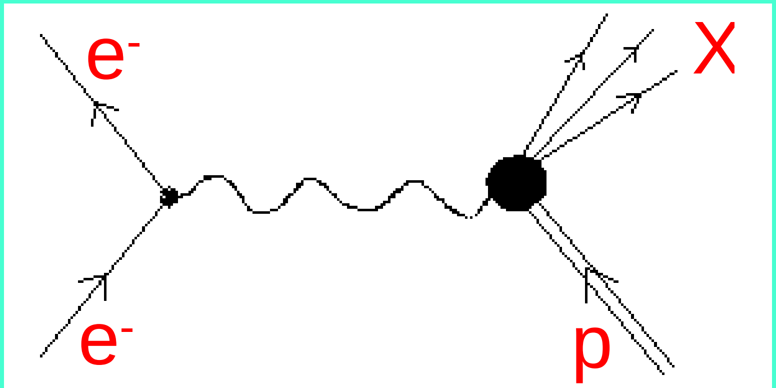
Phenomenological level , if not from first principles?

To that end , we go the regime of Small

Bjorken x of Deep Inelastic Scattering.

Deep Inelastic Scattering (DIS) :

e – p scattering : e p $\rightarrow$ e X



Q^2 = Virtuality of the virtual photon

$$Q^2 = 4EE' \sin\theta/2$$

x = Fraction of momentum carried by the parton stuck

by the virtual photon with $0 < x < 1$ (Bjorken-x)

$$x = \frac{Q^2}{2M(E - E')}$$

Small Bj x, ‘ Wee partons’

x~0.

In this regime, the behaviour of the sea quark densities is driven by gluon emissions and splittings . The deeper the proton structure is probed,more and more gluon interactions can be observed.

These gluons may follow self similar pattern which should be menifested through power laws in x at fixed Q^2:and in Q^2 at fixed x.

Figure shows the logarithm of the un-integrated u-quark density as a function logx(Fig a)and as a function of log Q^2(Figb) for fixed Q^2.(Q^2=10Gev^2)and fixed x (x=0.001) respectively. The full and dashed lines correspond to GRV

Parametrization in LO and NLO respectively

It suggests that x and Q^2 (or their suitable

Functions) can be treated as magnification factors and the nucleon structure functions exhibit self-similarity at small Bjorken x.

=>motivation for fractal inspired models.

For x ≤ 0.01 i.e. below the valence quark region, the unintegrated u-quark density function exhibits a linear behavior as a function of Q^2 for fixed x and as a function of x for fixed Q^2.

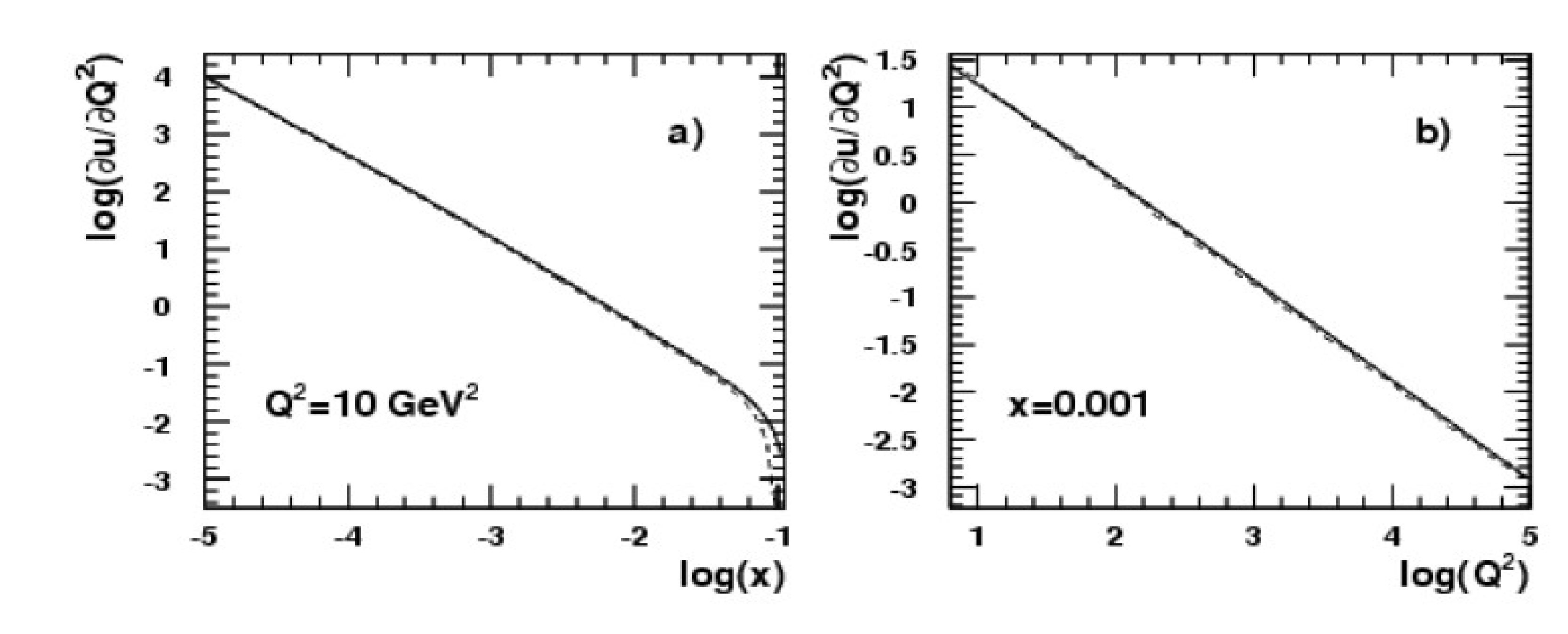


Figure : Logarithm of the un-integrated u-quark density (Lastovicka 2002).

FRACTAL INSPIRED MODEL OF PROTON STRUCTURE FUNCTIONS

- To formulate such model:to choose proper magnification factors
- They should be positive ,non-zero and dimensionless.
- Lastovicka(2002):$M_1=1/x;M_2=1+Q^2/Q_0^2$
- Un-integrated parton density of flavor i:

$$\mathrm{Log}\ f_i(x,k^2_T) = D^i_1 \log 1/x \log(1+Q^2/Q_0^2) + D^i_2 \log 1/x + D_3^i \log(1+Q^2/Q_0^2) + D_0^{\ i}$$

where D_0^i is the normalization constant, D_1 is the dimensional correlation relating the two magnification factors. D_2 is the fractal dimension associated with the magnification factor $M_{1.}$ D_3 is the fractal dimension associated with the magnification factor M_2.

The integrated parton density is given by,

$$q_i\left(x, Q^2\right) = \int_0^{Q^2} f_i\left(x, k_T{}^2\right) dk_T{}^2$$

Using the definition of structure function as,

$$F_2\left(x, Q^2\right) = x \sum e_i^2 \left(q_i\left(x, Q^2\right) + \overline{q_i}\left(x, Q^2\right)\right)$$

The structure function can be obtained as,

$$F_2\left(x,Q^2\right)=\frac{\exp\left(D_0{}^q\right)Q_0^2x^{-D_2+1}}{1+D_3{}^q+D_1\log 1/x}\left[x^{-D_1{}^q\log\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{D_3{}^q+1}-1\right]$$

The parameters are,

$D_0 = 0.339\pm0.145$

$D_1=0.073\pm0.001$

$D_2=1.013\pm0.01$

$D_3=-1.287\pm0.01$

$Q_0{}^2=0.062\pm0.01$

Expected properties of Fractal Dimensions:

$D_1, D_2, D_3 \geq 0.$

But Lastovicka's set => $D_3 < 0.$

Additional Problem:

There is a singularity at $x \approx 0.019$ well within the HERA range($0.2 \geq x \geq 6.2 \times 10^{-7}$)where

$$1 + D_3 + D_1 \log(1/x) = 0.$$

Can these problems be avoided?

A NEW PARAMETERISATION OF

PROTON STRUCTURE FUNCTION

WITH SELF SIMILARITY (MODEL-II)

****

We take the magnification factors as

$$\frac{1}{x} \quad \text{and} \quad \frac{Q_0^2}{Q_0^2+Q^2}$$

Then we can get the unintegrated parton density as,

$$\log f_i(x,Q^2) = D_1 \log\frac{1}{x}\log\left(\frac{Q_0^2}{Q_0^2+Q^2}\right) + D_2\log\frac{1}{x} + D_3\log\left(\frac{Q_0^2}{Q_0^2+Q^2}\right) + D_0^i$$

The structure function is given by,

$$F_2\left(x,Q^2\right) = \frac{\exp\left(D_0\right)Q_0^2 x^{-D_2+1}}{1-D_3-D_1\log x}\left[x^{D_1\log\left(1+\frac{Q^2}{Q_0^2}\right)}\left(1+\frac{Q^2}{Q_0^2}\right)^{-D_3+1}-1\right]$$

Fit	D_0	D_1	D_2	D_3	Q_0^2	χ^2	χ^2/dof
All Fit	0.6345 ± 0.0145	0.2398 ± 0.0125	1.2581 ± 0.0157	1.4352 ± 0.0113	0.0498 ± 0.0013	272.743	1.356
D_1 fixed	0.4961 ± 0.041	0	1.2009 ± 0.039	1.4279 ± 0.0584	0.0427 ± 0.0039	142.281	0.071

The fitted parameters using Model - II

In this D_1,D_2,D_3 are all positive

&

$1-D_3-D_1\log 1/x=0$,

Only at x, outside the physical region

$0\leq x\leq 1$.

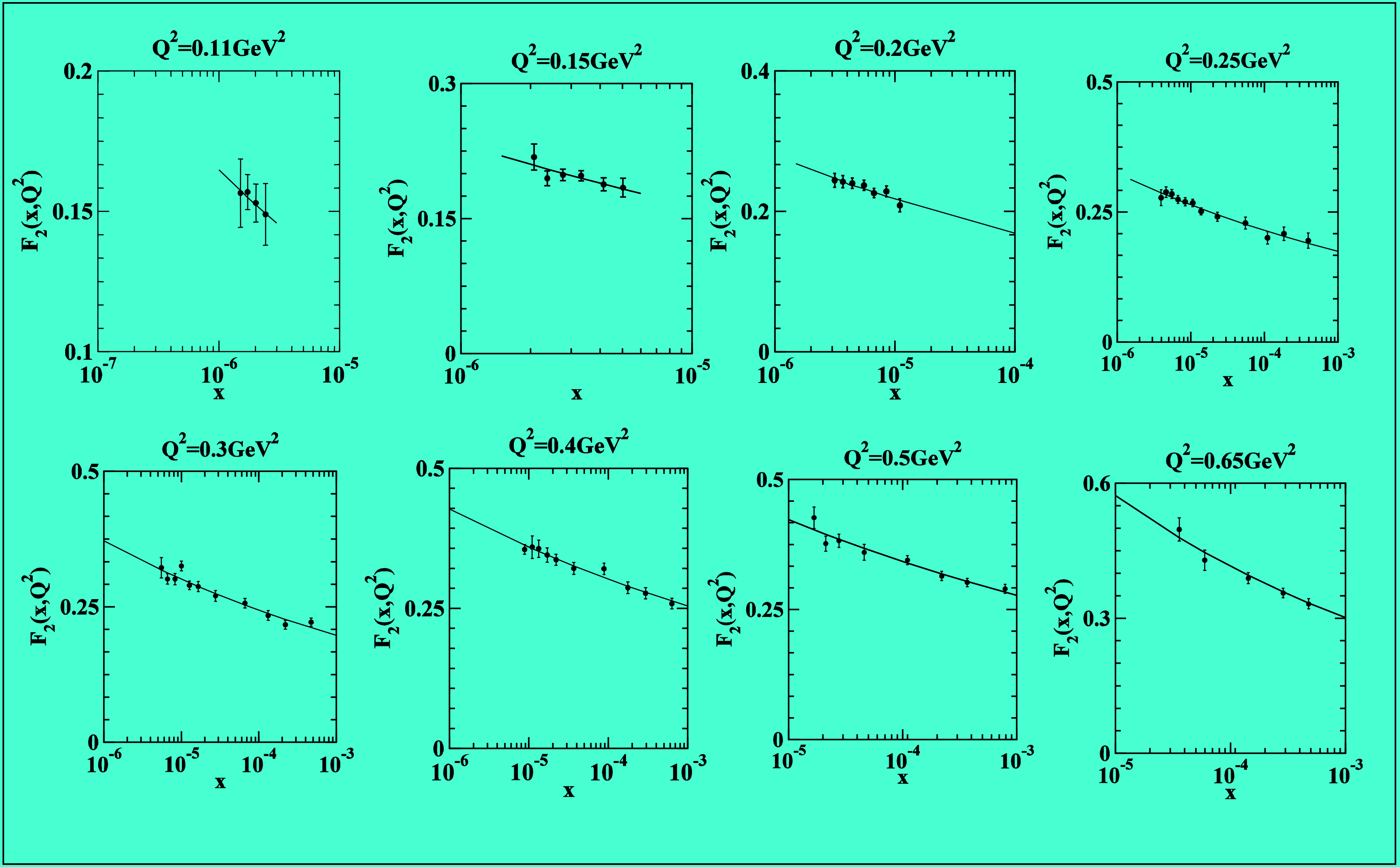
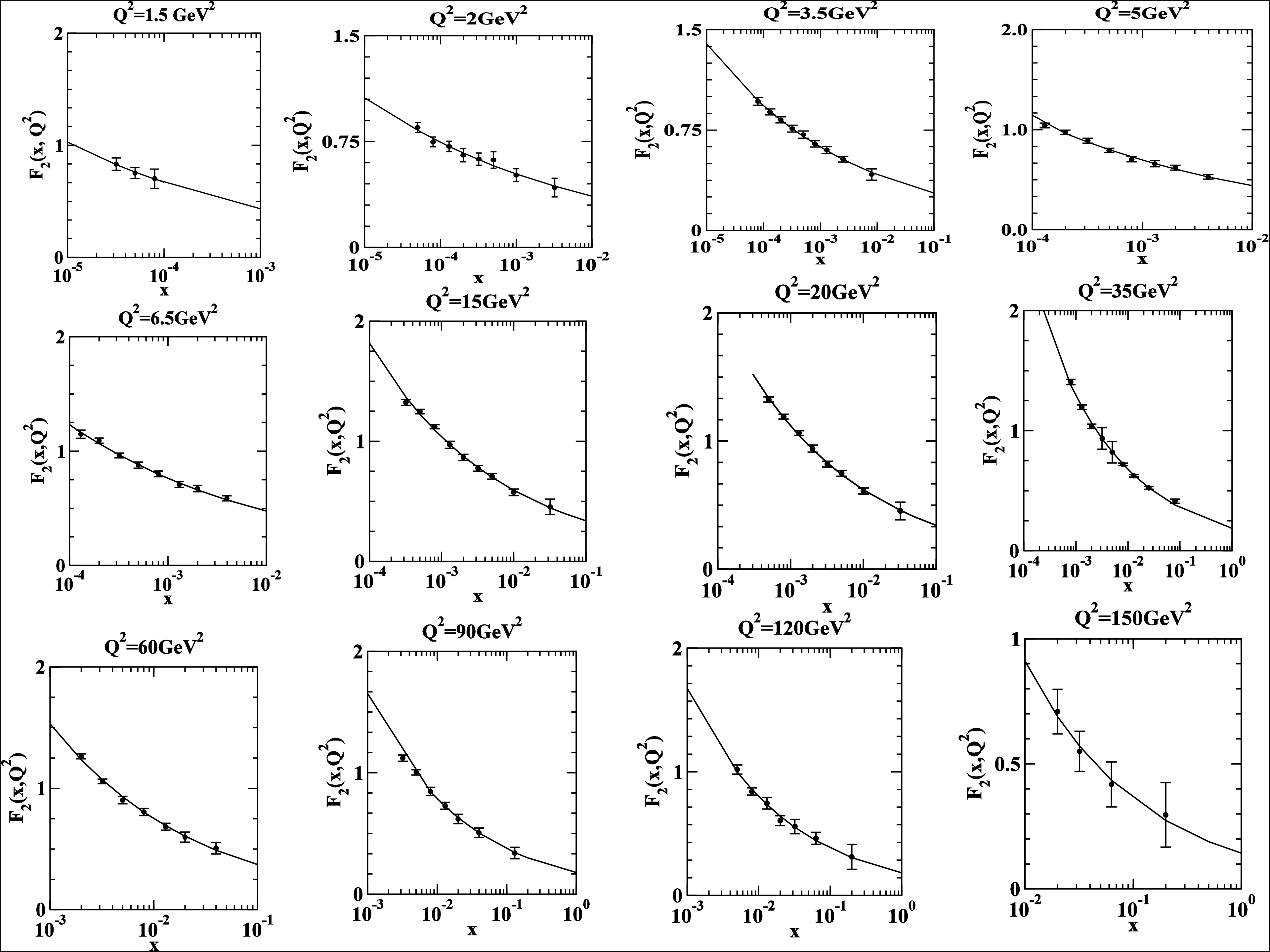


Fig. $F_2(x,Q^2)$ versus x in bins of Q^2 with $D_1 = 0$


Q^2=1.5 GeV^2
Q^2=2GeV^2
Q^2=3.5GeV^2
Q^2=5GeV^2
Q^2=6.5GeV^2
Q^2=15GeV^2
Q^2=20GeV^2
Q^2=35GeV^2
Q^2=60GeV^2
Q^2=90GeV^2
Q^2=120GeV^2
Q^2=150GeV^2
F$_2$(x, Q^2)
x

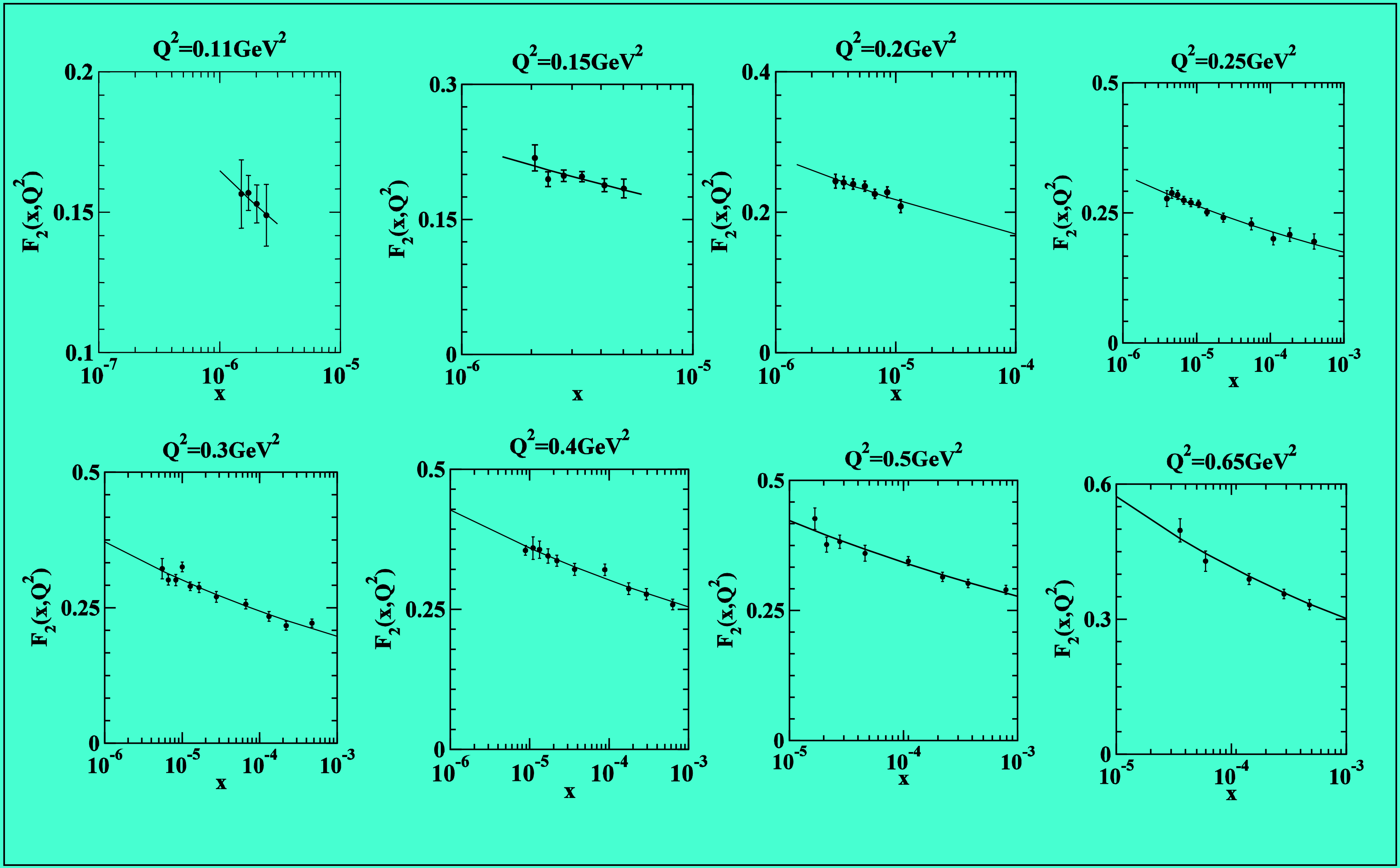
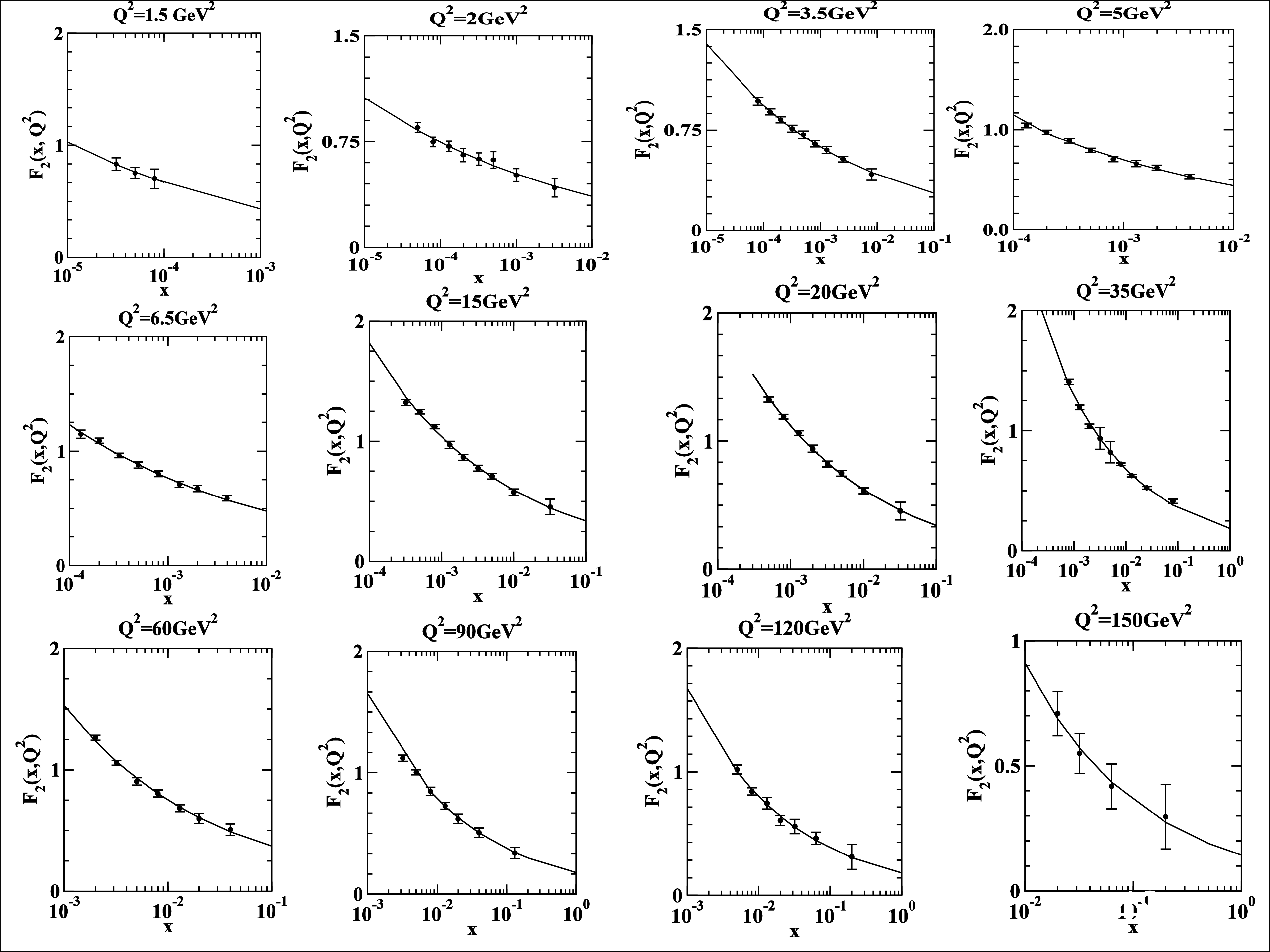


Fig. $F_2(x,Q^2)$ versus x in bins of Q^2 with $D_1 \neq 0$



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Q^2=15GeV^2
Q^2=20GeV^2
Q^2=35GeV^2
Q^2=60GeV^2
Q^2=90GeV^2
Q^2=120GeV^2
Q^2=150GeV^2
F_2(x, Q^2)
x

Fractal inspired models of gluon densities at small x:

The forms of unintegrated and integrated gluon densities with self-similarity are identical to those of the quark distribution except the set of parameters will be different.

Thus form of unintegrated gluon density $g(x,q^2)$ will be,

Model I

$$\log g\left(x,q^2\right) = D_1 \log \frac{1}{x} \log\left(\frac{Q_0^2 + k_T^2}{Q_0^2}\right) + D_2 \log \frac{1}{x} + D_3 \log\left(\frac{Q_0^2 + k_T^2}{Q_0^2}\right) + D_0^{ig} \quad .$$

Model II

$$\log g\left(x,q^2\right) = D_1^g \log \frac{1}{x} \log\left(\frac{Q_0^2}{Q_0^2 + q^2}\right) + D_2^g \log \frac{1}{x} + D_3^g \log\left(\frac{Q_0^2}{Q_0^2 + q^2}\right) + D_0^g \quad .$$

Using relation,

$$g\left(x,Q^2\right)=\int_0^{Q^2} g\left(x,k_T{}^2\right)dk_T{}^2$$

We get the following forms of gluon distribution,

Model I

$$xg^{I}(x,Q^2)=\frac{e^{D_0^g}Q_0^2x^{-D_2^g+1}}{1+D_3^g+D_1^g\log {}^{1}\!/\!_{x}}\left[x^{-D_1^g\left(1+{}^{Q^2}\!/\!_{Q_0^2}\right)}\left(1+{}^{Q^2}\!/\!_{Q_0^2}\right)^{D_3^g+1}-1\right]$$

Model II

$$xg^{II}(x,Q^2)=\frac{e^{D_0^g}Q_0^2x^{-D_2^g+1}}{1-D_3^g-D_1^g\log {}^{1}\!/\!_{x}}\left[x^{D_1^g\left(1+{}^{Q^2}\!/\!_{Q_0^2}\right)}\left(1+{}^{Q^2}\!/\!_{Q_0^2}\right)^{-D_3^g+1}-1\right]$$

We need four parameters D_0^g, D_1^g, D_2^g and D_3^g to be determined.

We choose MRST04LO solutions for comparison. A comparison of the models with MRST04LO exact results gives the following set of parameters,

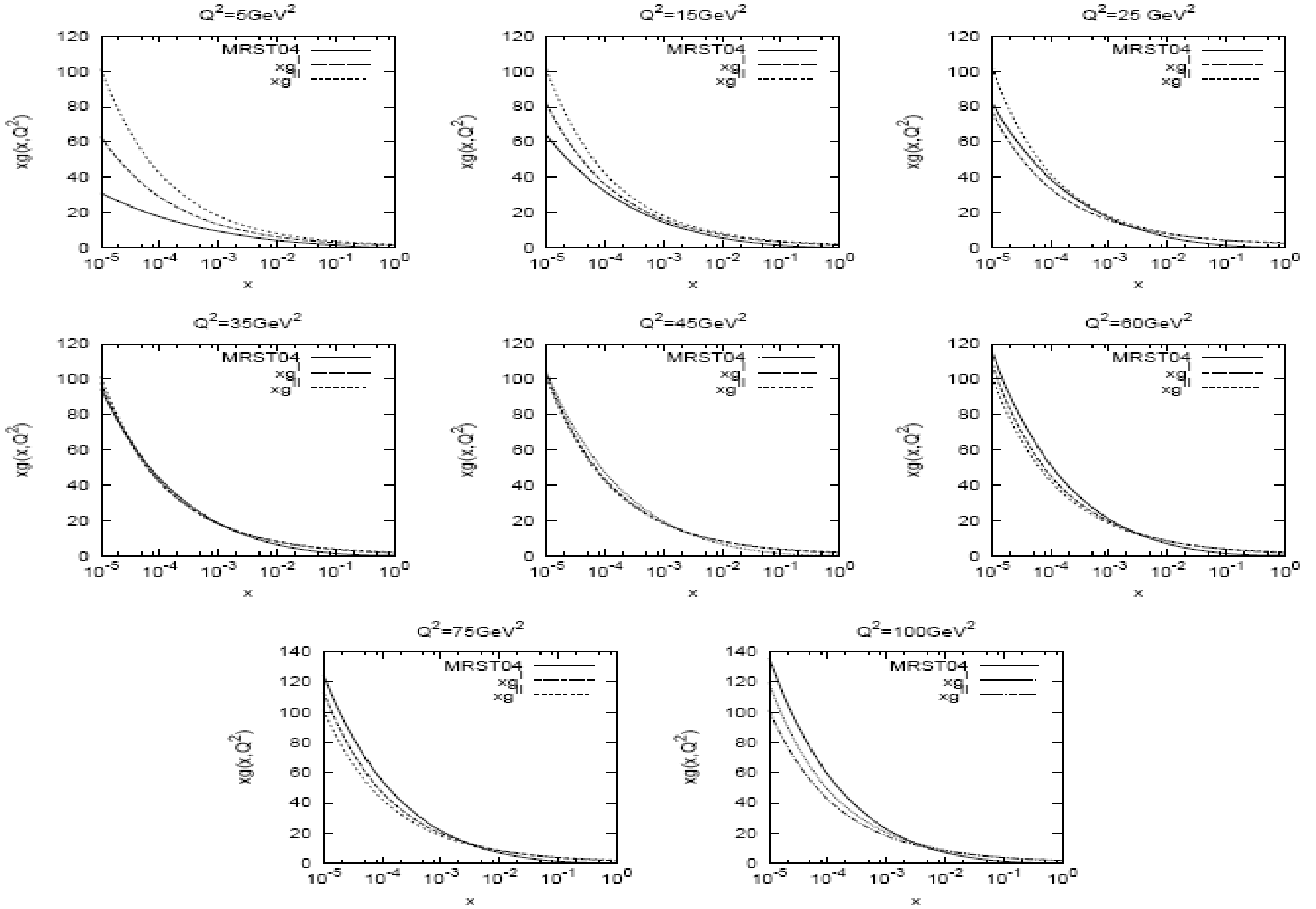
Model I	Model II
$D_0^g = 2.1961$,	$D_0^g = 2.9594$
$D_1^g = 0.073$	$D_1^g = .2398$
$D_2^g = 1.2662$	$D_2^g = 1.4484$
$D_3^g = -1.287$	$D_3^g = 1.4352$
$Q_0^2 = 0.062$	$Q_0^2 = .049$

Comparison with the results obtained with quark distribution :

$D_0^q \neq D_0^g$

$D_2^q \neq D_2^g$ and $D_2^g > D_2^q$, which conforms to the feature that for any Q^2, gluon is steeper than quark density.

We have also obtained the gluon density using the magnification factor used by Lastovicka (Model I) with negative D^g_3 . For Model II for Gluon, all fractal parameters are positive once again.



Q²=5GeV²
Q²=15GeV²
Q²=25 GeV²
Q²=35GeV²
Q²=45GeV²
Q²=60GeV²
Q²=75GeV²
Q²=100GeV²
MRST04
xgI
xgII
xg(x,Q²)
x

Compatibility of the models with QCD evolution equations

Gluon density is measured from the slope of the structure function according to DGLAP equations based equalities.

Prytz formula:

$$G(2x) \approx \frac{9\pi}{5\alpha_S} \frac{3}{2} \frac{dF_2}{d \log Q^2}$$

K. Bora, DKC

$$G(4x/3) \approx \frac{3\pi}{5\alpha_S} \frac{dF_2}{d \log Q^2}$$

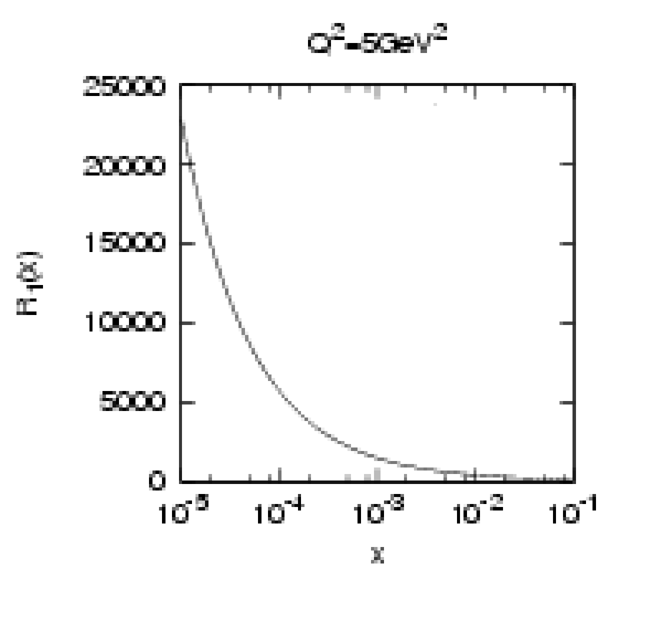
Gay Ducati *et al*

$$G\left(\frac{x}{1-\alpha} (3/2-\alpha) \right) = \frac{27\pi}{10\alpha_S} \frac{dF_2}{d \log Q^2}$$

Compatibility of fractal model with QCD evolution based equations

We define the ratio,

$$R_1 = \frac{G(2x, Q^2)}{\frac{27\pi}{10\alpha_S}\frac{\partial F_2}{\partial \log Q^2}}$$



R_1 versus x graph

Prytz relation is modified within the fractal model.

$$G(2x) \approx x^{0.6434} \frac{9\pi}{5\alpha_S} \frac{3}{2} \frac{dF_2}{d \log Q^2}$$

A similar modification also occurs for other realtions as well.

Further applications of the model :

(1) Deep Inelastic Neutrino Scattering with CCFR Data at GeV Scale

(2) UHE Neutrinos : Testing Self Similar Quarks at PeV Scale

(3) Proton proton collisions at LHC : Testing Self Similar Gluons at TeV Scale

DEEP INELASTIC NEUTRINO SCATTERING

CCFR DIS Neutrino Experiments have not yet reached small x regime:

$0.0075 < x \leq 0.75$ to be compared with HERA: $x \approx 0.00000062$.

We test the validity or otherwise of the self similar models even at such large x regime:

Figures of F_2 and xF_3 for Models I and II:

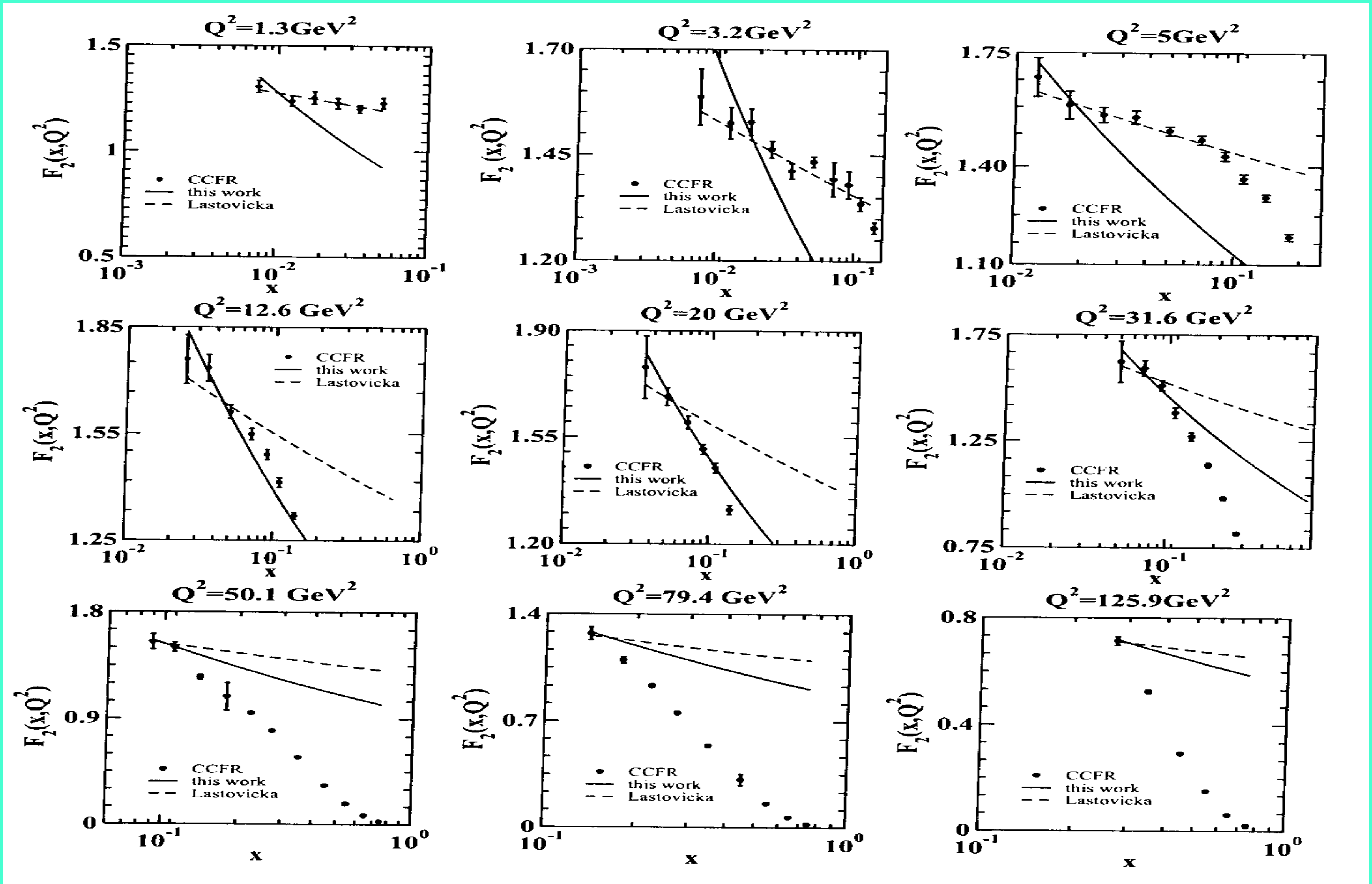


Fig. $F_2(x,Q^2)$ vs x in bins of Q^2 with $D_1 = 0$

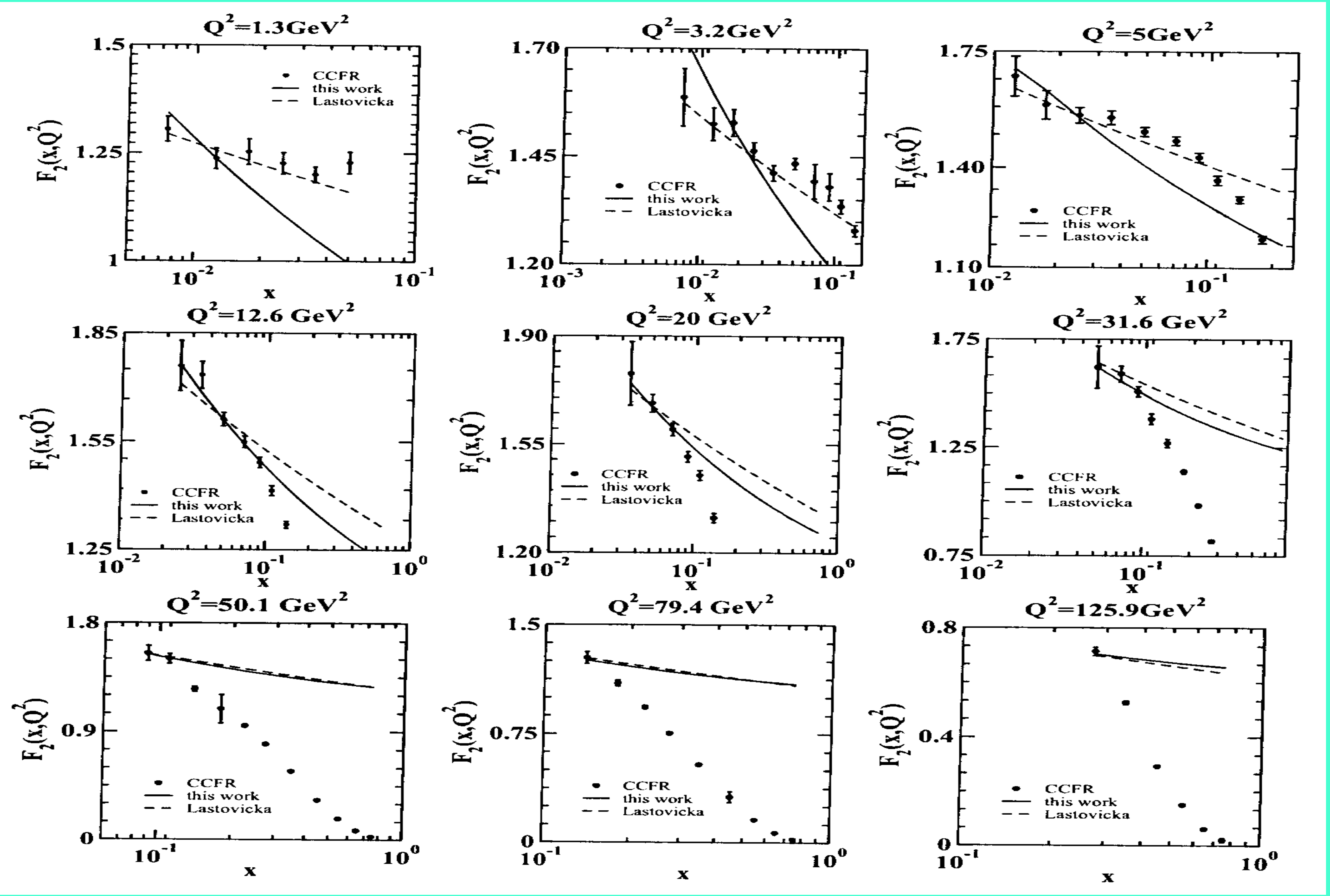


Fig. $F_2(x,Q^2)$ vs x in bins of Q^2 with $D_1 \neq 0$

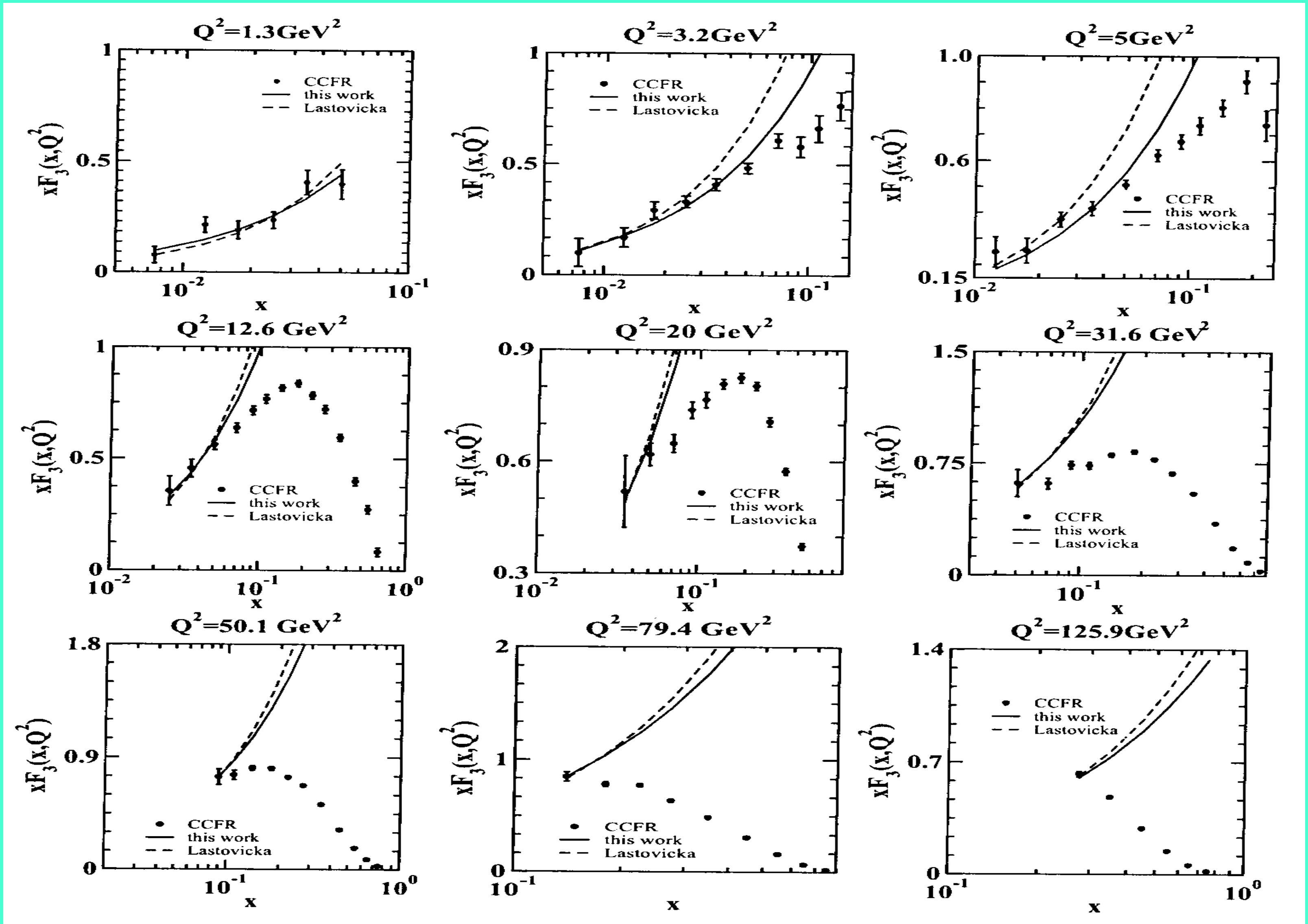


Fig. $xF_3(x,Q^2)$ vs x in bins of Q^2 with $D_1 = 0$

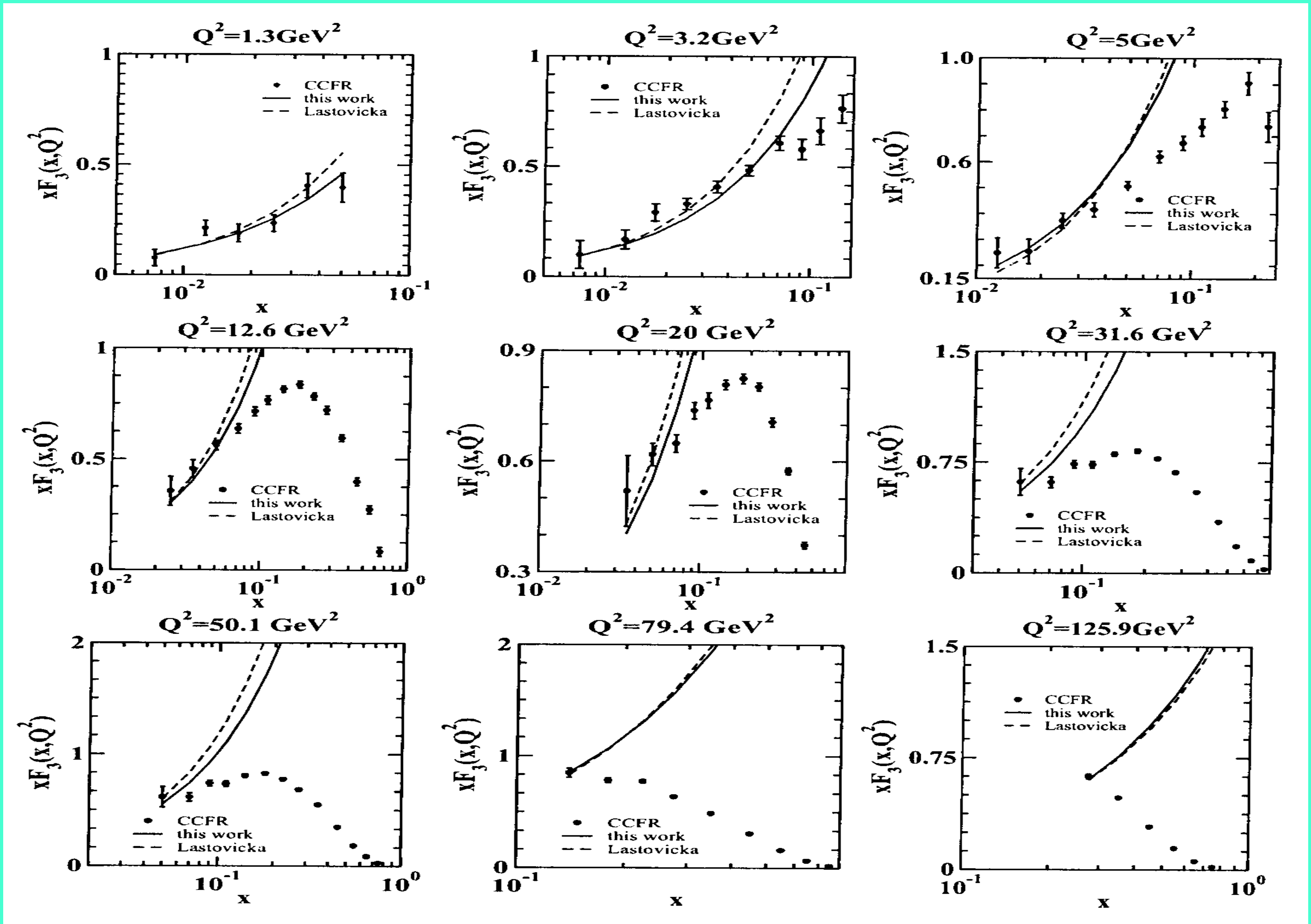


Fig. $xF_3(x,Q^2)$ vs x in bins of Q^2 with $D_1 \neq 0$

Main Features:

For large x, both the models overshoot CCFR data.

An additional large x multiplicative factor ($\alpha \approx 0.2$-1.2) needed: $(1-x)^{\alpha}$

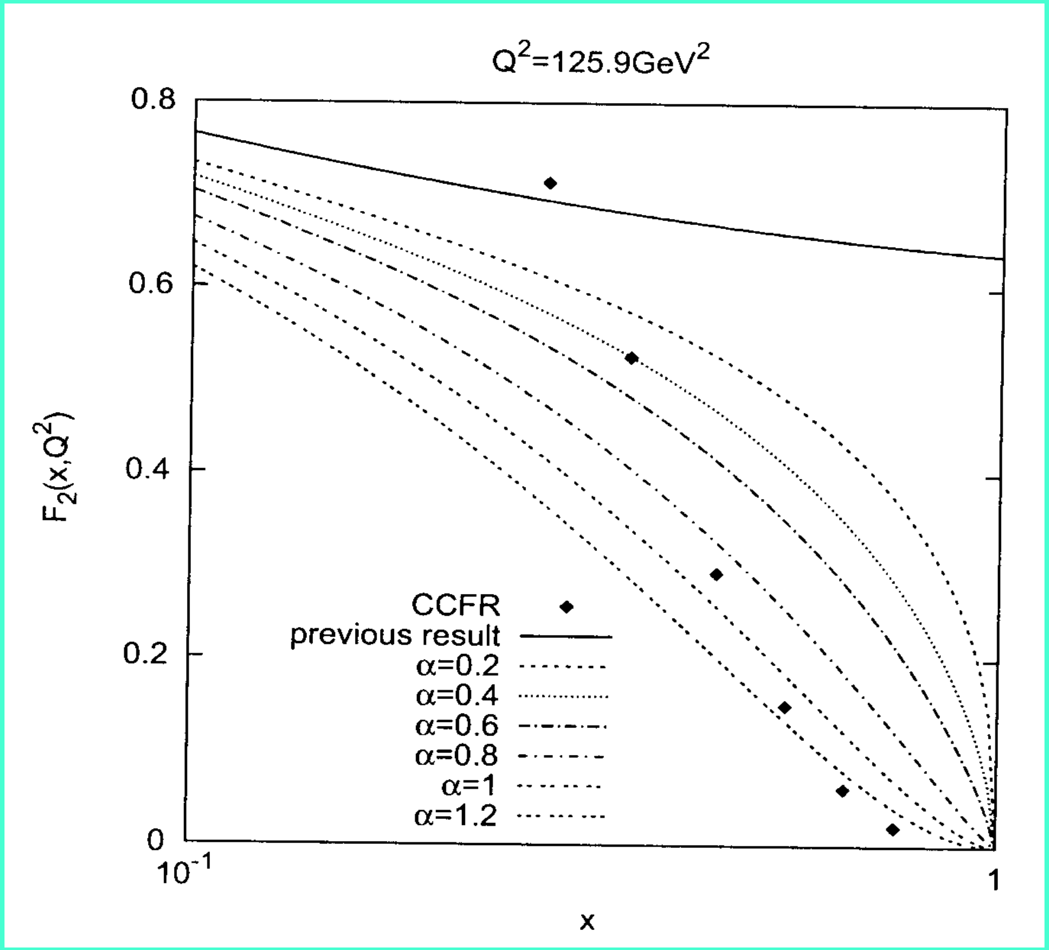


Fig. F_2 is plotted as a function of x at Q^2 = 125.9 GeV^2 with large x damping effect

ULTRA HIGH ENERGY NEUTRINO NUCLEON SCATTERING

Physics with UHE Neutrinos ($E_\nu > 10^7$) below and above GZK cut-off 6×10^{10} GeV is a topical area of research.

Neutrino projects :Pierre Auger Observatory (PAO),ICE CUBE,AMANDA,NESTOR,ANTARES are designed to detect them.

Several collaborations likeFly"s Eye,AGA-SA and NESTOR imposed upper bounds on the neutrino flux and turned them into Upper bounds on neutrino nucleon cross-sections.

Such cross-sections probe ultra-small x:

Typical x $\sim M^2_W/(2M_N E_\nu)$

$x \sim 3.2 \times 10^{-9}$ for $E_\nu \sim 10^{12}$ GeV

Self Similarity/fractals should make sense.

- Neglecting xF_3 and F_L, we have made calculations for total neutrino-nucleon cross-sections with Fractal Inspired Model –II.
- For $10^4 \leq E_\nu \leq 10^{12}$ G e v, results are represented by simple power laws:
- $б_{cc} = 3x10^{-36} cm^2 (E_\nu / 1Gev)^{0.2753}$
- $б_{nc} = 9x10^{-37} cm^2 (E_\nu / 1Gev)^{0.2512}$

$$\sigma_{tot} = 6\times10^{-36} cm^2 (E_\nu / 1Gev)^{0.2704}$$

These results are well within the model independent upper bounds of neutrino nucleon Cross-sections imposed by RICE and AGASA Collaborations at $E=10^{10}$, $10^{10.5}$ and 10^{11} G e v

E_ν (Gev)	Prediction (cm^2)	Experimental Bound (cm^2)
10^{10}	*0.315×10^{-31}*	$<$*1.2×10^{-29}*
$10^{10.5}$	0.521×10^{-31}	$<0.36 \times 10^{-29}$
10^{11}	0.728×10^{-31}	$<0.38 \times 10^{-29}$

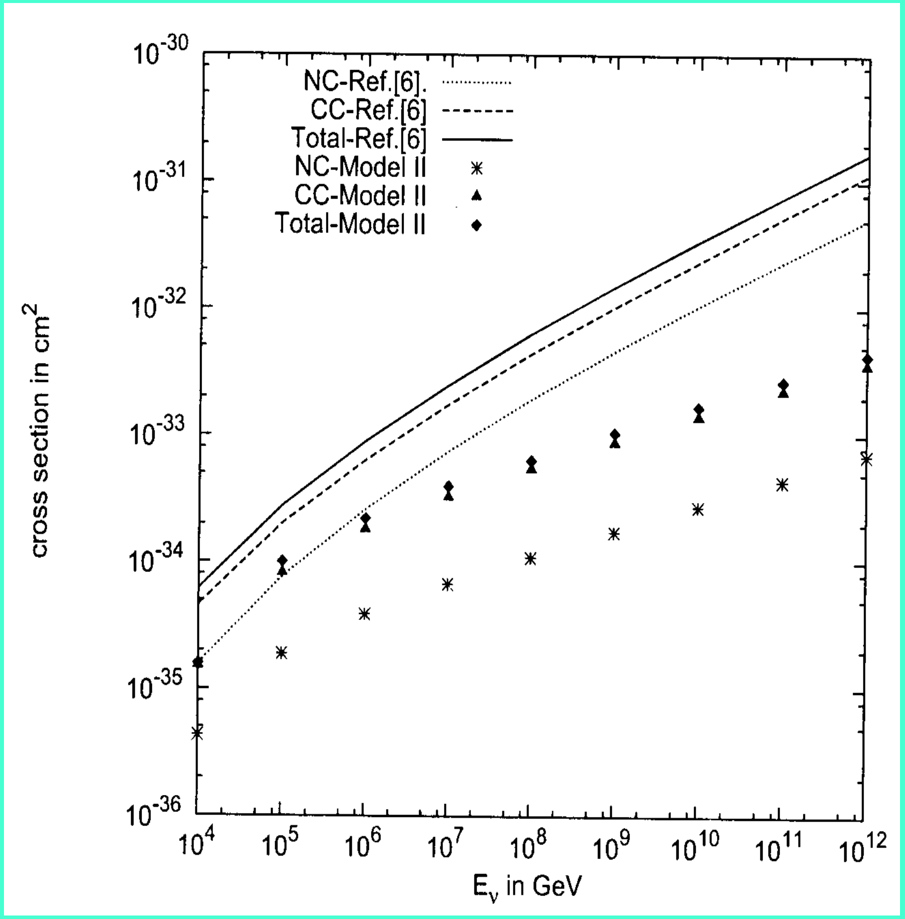


Fig. Results of model II

SUMMARY :

Assuming the validity of extrapolation of Self-similar parton densities from HERA (GeV Scale) to UHE(PeV ~10^{12} GeV), predictions are not inconsistent with available neutrino nucleon б bounds

PREDICTIONS AT LHC

Consider Inclusive production like

$$P+P \rightarrow \pi^0 + X$$

at LHC with E_{CM} =14TeV

There is a large overlap in x at LHC with the x-range covered at HERA. Details of the

Parton kinematics at LHC depends on the on the invariant mass M of the final state

$M=[Q^2]^{1/2}$ and the rapidity y. The dominant

Partons in the two protons are:

$X_{1,2}$ ~M/(14 TeV).exp(±y)

For low M, and large y,

Small x regime ,$x_{1,2}$~10^{-6} achievable

Fractal Inspired quark and gluon distributions applicable.

Work in Progress

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THANKS