QCD: strings and no strings

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Lattice simulations

Strings and fluxtubes

Breaking strings

Effective bosonic strings

Baryonic string configurations

Outlook

PPISR, Bidalur, Bangalore, 14.1.09

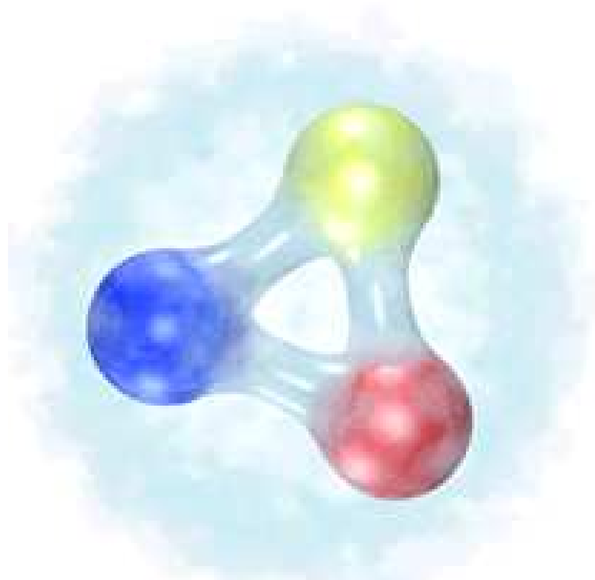
QCD (Theory of strong interactions)

$$\mathcal{L}_{QCD} = -\frac{1}{16\pi\alpha_s}FF + \bar{\psi}_f(\not{D} + m_f)\psi_f$$

$\longrightarrow$ asymptotic freedom: $\alpha_s(q) \stackrel{q\to\infty}{\longrightarrow} 0$

$\stackrel{?}{\longrightarrow}$ confinement

$\stackrel{?}{\longrightarrow}$ chiral symmetry breaking

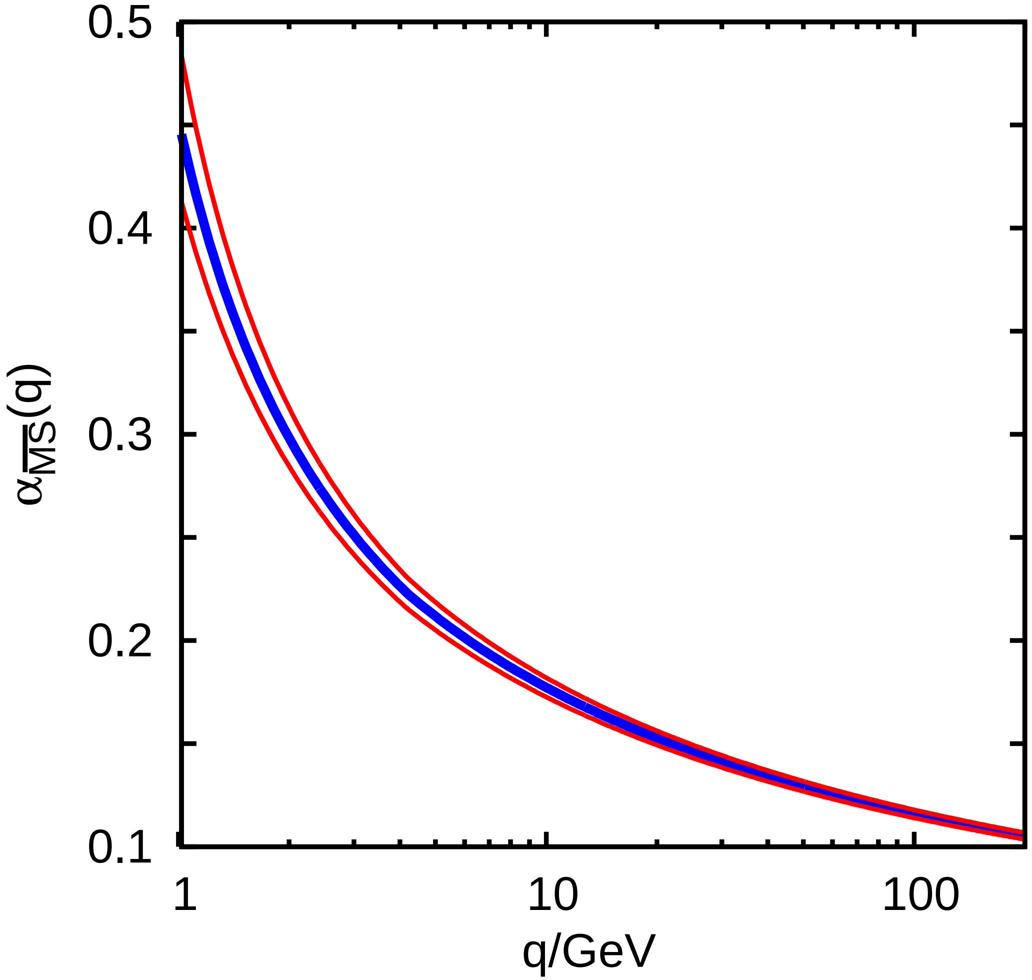


proton (artist's impression)

Theoretically beautiful but quantitative predictions cause big headache in the region dominated by *strong* QCD !

$\Longrightarrow$ computer simulation

The running coupling



$\alpha_{\overline{\mathrm{MS}}}(m_Z) = 0.1174(20) \longrightarrow$

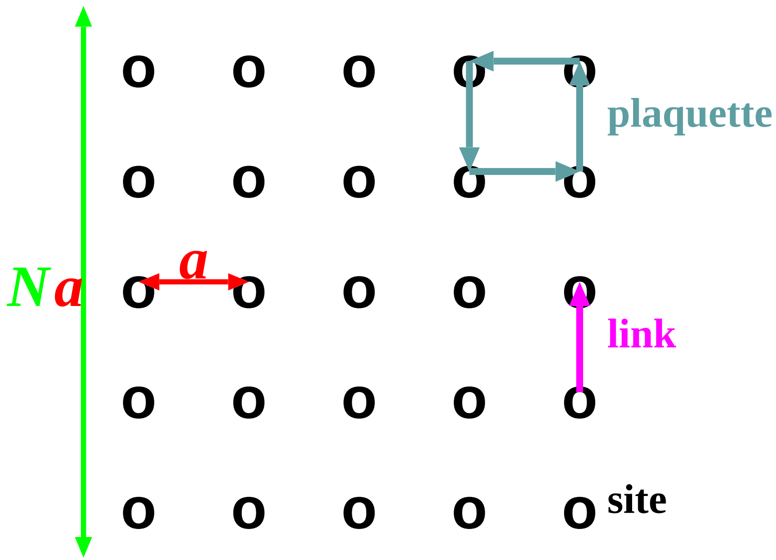
$\alpha_{\overline{\mathrm{MS}}}(1.5\ \mathrm{GeV}) = 0.346^{+22}_{-21}$

6.3 % error at 1.5 GeV $\longrightarrow$

1.7 % at 91 GeV !

$m_p = 0.93827203(8)$ GeV !!!

Lattice Gauge Theory



typical values:
$a^{-1} = 1.5$–4 GeV, $Na = 1.5$–6 fm

continuum limit: $a \to 0$, Na fixed

infinite volume: $Na \to \infty$

$$\langle O \rangle = \frac{1}{Z} \int [dU]\, [d\psi][d\bar{\psi}]\, O[U] e^{-S[U,\psi,\bar{\psi}]}$$

"Measurement": averaging over a *representative* ensemble of gluon-configurations $\{U_i\}$ with probability $P(U_i) \propto \int [d\psi][d\bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$

$$\langle O \rangle = \frac{1}{n} \sum_{i=1}^{n} O(U_i) + \Delta O \qquad \Delta O \propto \frac{1}{\sqrt{n}} \xrightarrow{n\to\infty} 0$$

Input: $\mathcal{L}_{\mathcal{QCD}} = -\frac{1}{16\pi\alpha_L} FF + \bar{q}_f(\not{D} + m_f)q_f$

$$\begin{aligned} m_p^{\text{latt}} &= m_p^{\text{phys}} && \longrightarrow && a \\ m_\pi^{\text{latt}}/m_p^{\text{latt}} &= m_\pi^{\text{phys}}/m_p^{\text{phys}} && \longrightarrow && m_u \approx m_d \\ &\cdots \end{aligned}$$

Output: hadron masses, matrix elements, decay constants, etc...

Extrapolations:

1. $a \to 0$: functional form known.

2. $aN \to \infty$: harmless but often computationally expensive.

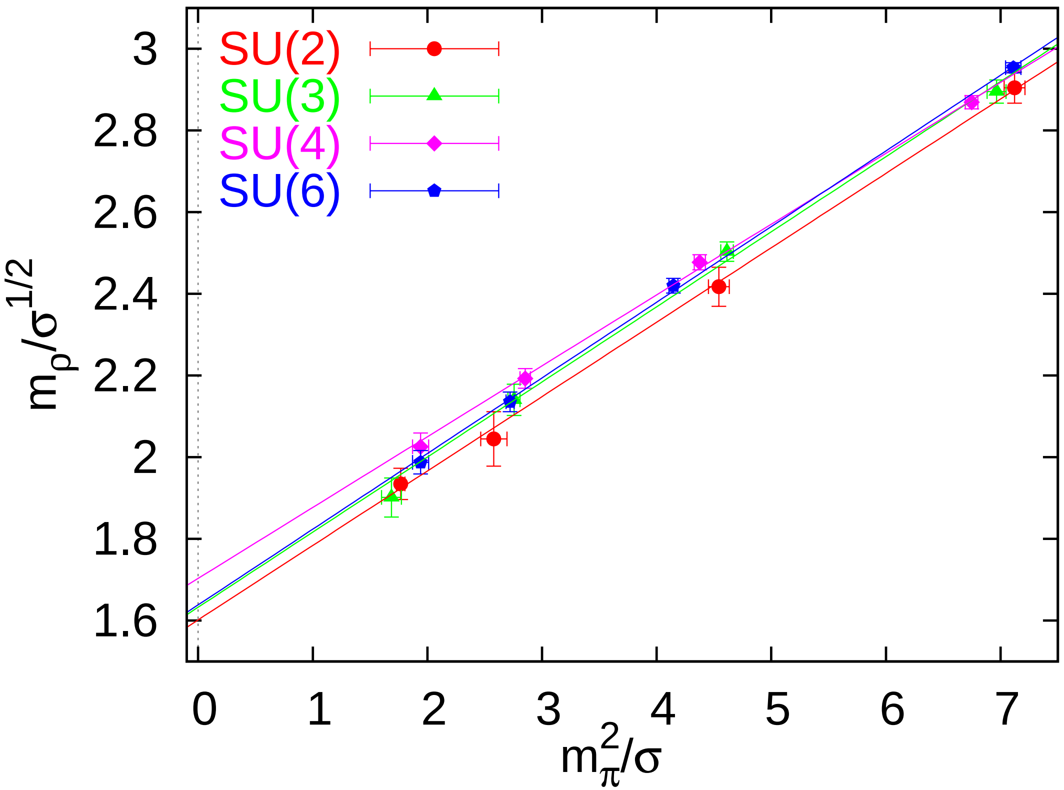
3. $m_q^{\text{latt}} \to m_q^{\text{phys}}$: chiral perturbation theory (χPT) but m_q^{latt} must be sufficiently small to start with (now physical m_π possible!).

Feature: create virtual worlds for other theorists:

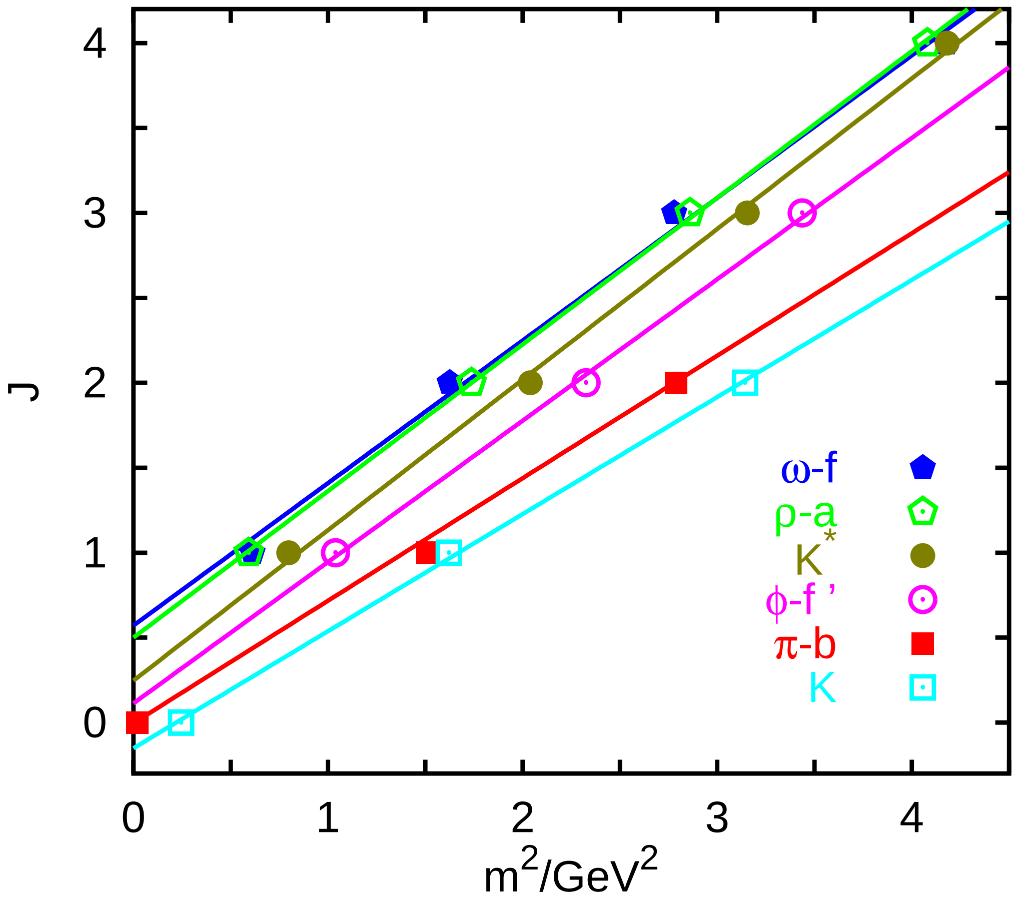
$\exists$ only one world that is accessible by experiment.

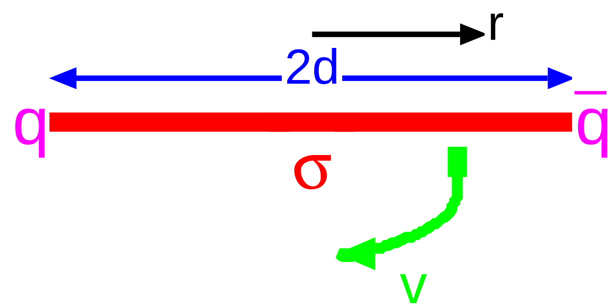
In simulations we can vary: T, n_f, N_c, m_q, $L = Na$ etc.

Mesons in $SU(N_c)$, $N_c = 2, 3, 4, 6$ F Bursa, GB 07/8, L Del Debbio et al 08



Excursion: Regge trajectories





string tension σ

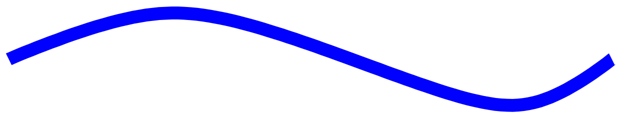
speed

$v(r) = r/d$

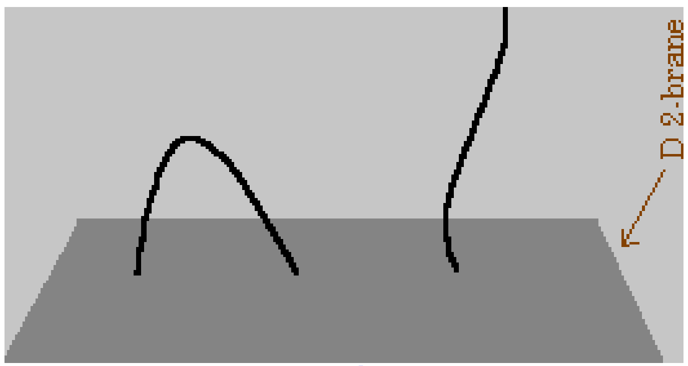
$$m = 2\int_0^d \frac{dr\,\sigma}{\sqrt{1-v^2(r)}} = \pi d\,\sigma \qquad J = 2\int_0^d \frac{dr\,\sigma r v(r)}{\sqrt{1-v^2(r)}} = \frac{\pi d^2 \sigma}{2} = \frac{1}{2\pi\sigma} m^2$$

Hadron physics pre-1973 ($\approx$ TOE post-2xyz ?)

open string

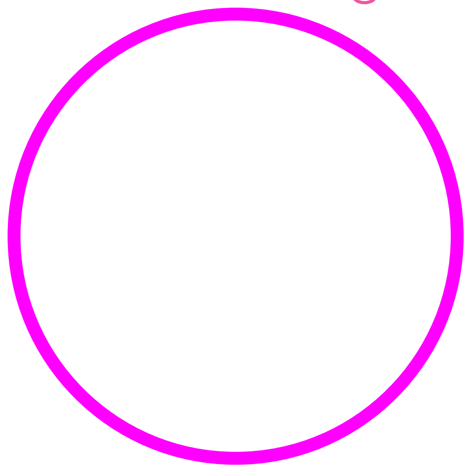


meson

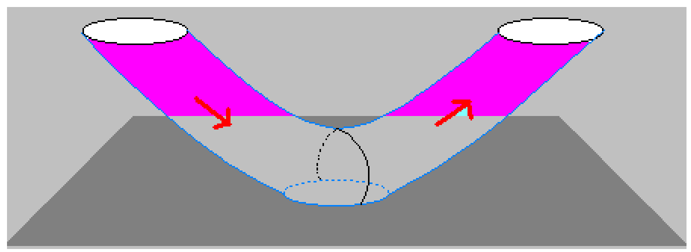


gauge theories

closed string

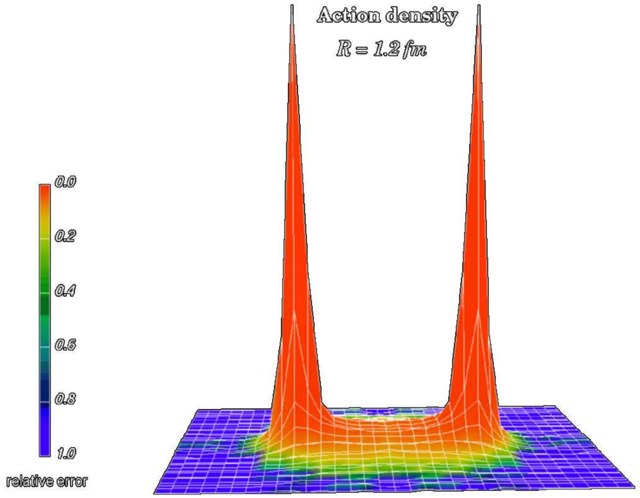


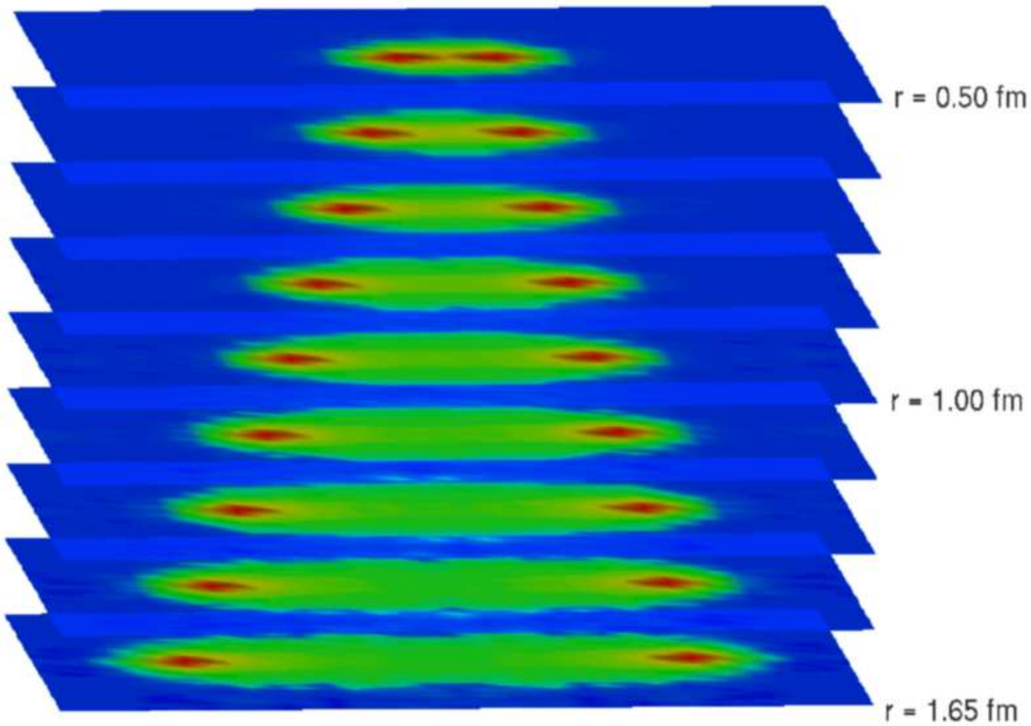
glueball



gravity

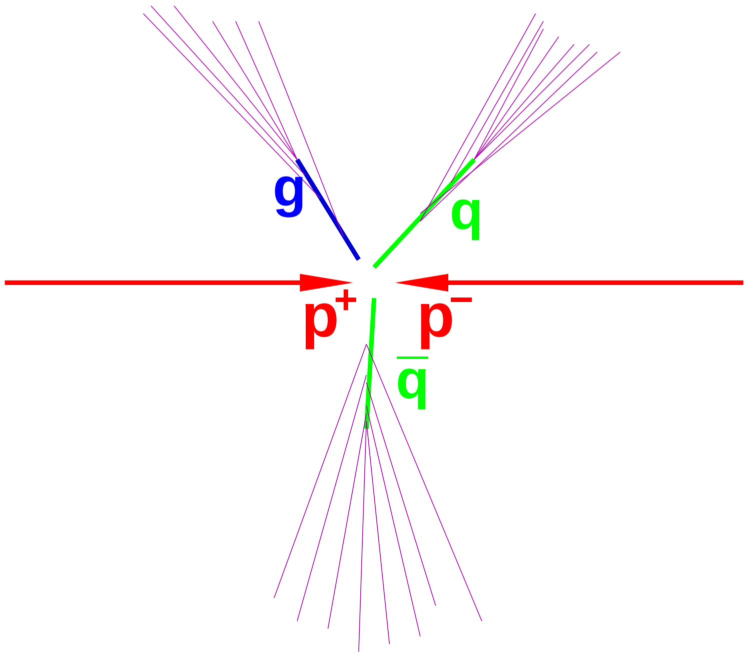
The QCD “string” 1995 GB, K. Schilling, C. Schlichter

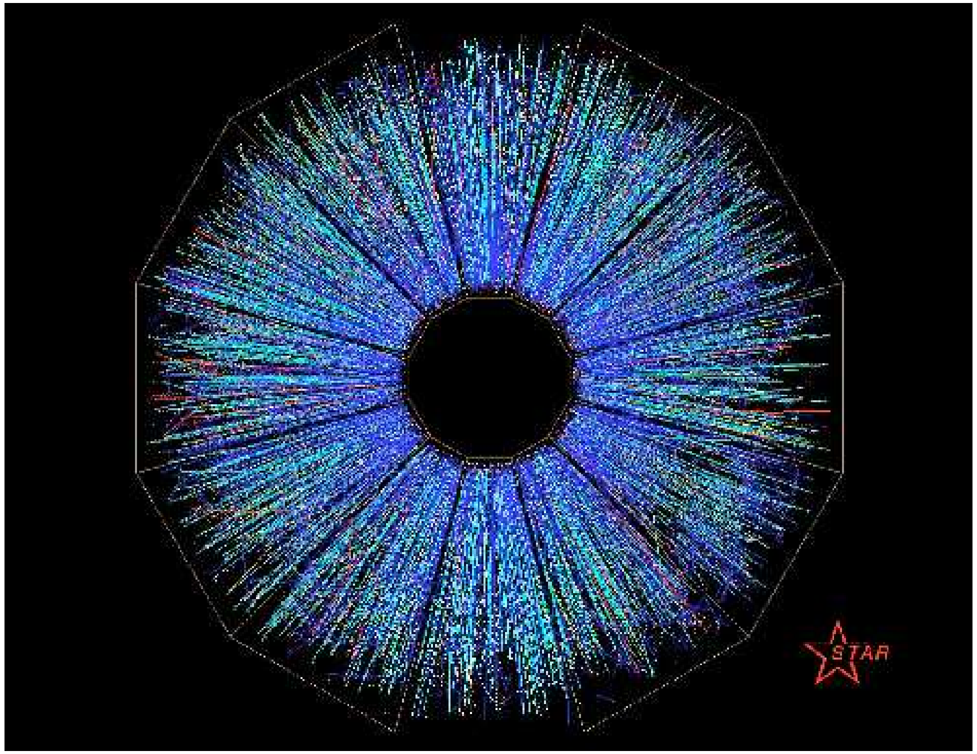




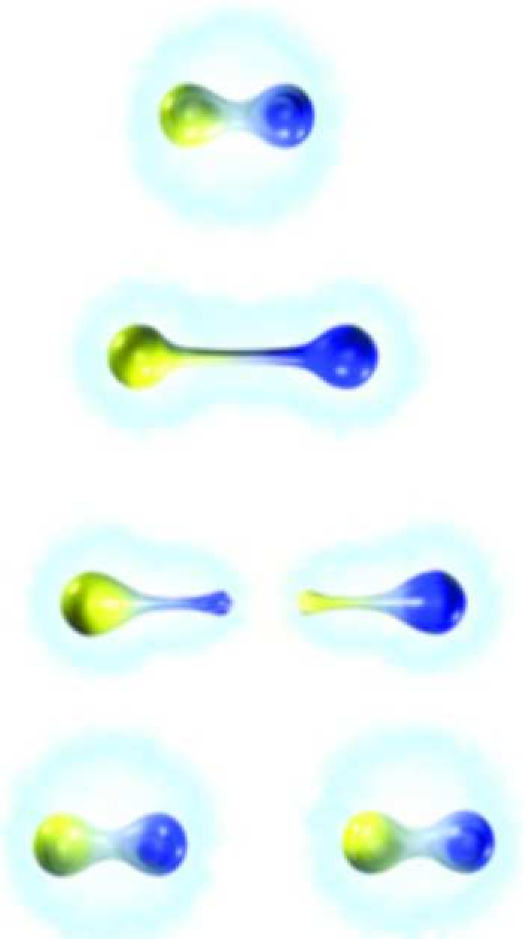
No sea quarks !!!

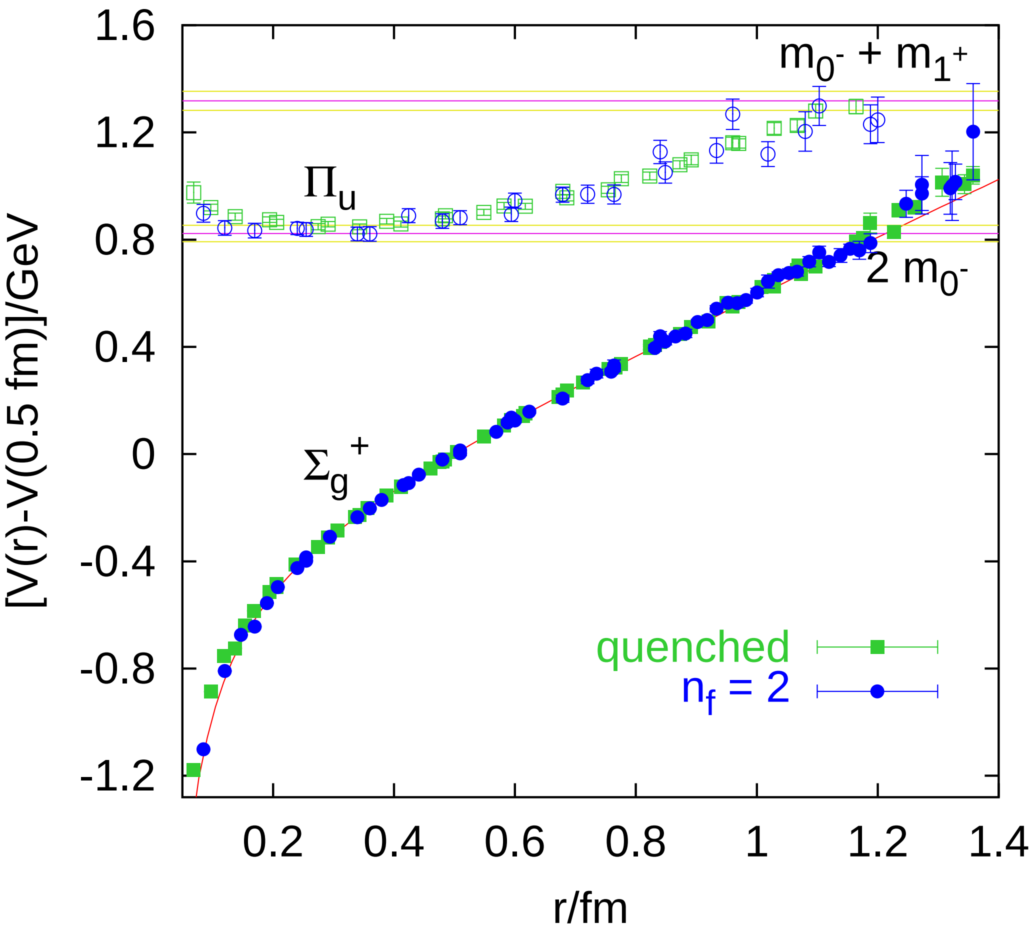
... and breaking the string





String breaking SESAM: GB et al. 98/99





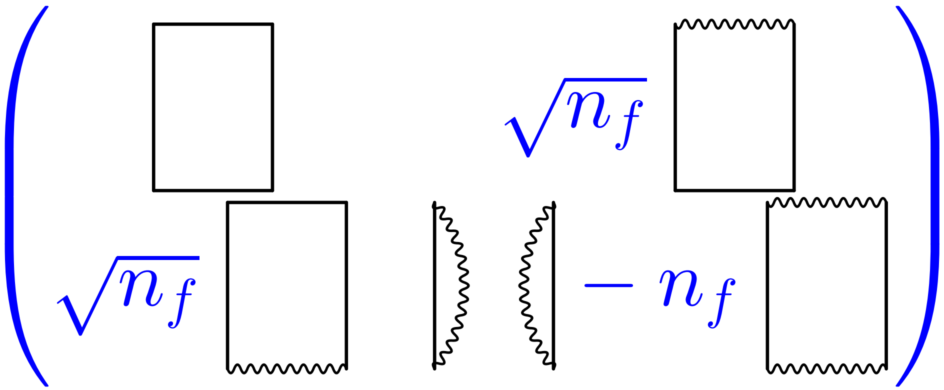
Two-state system:

Eigenstates:

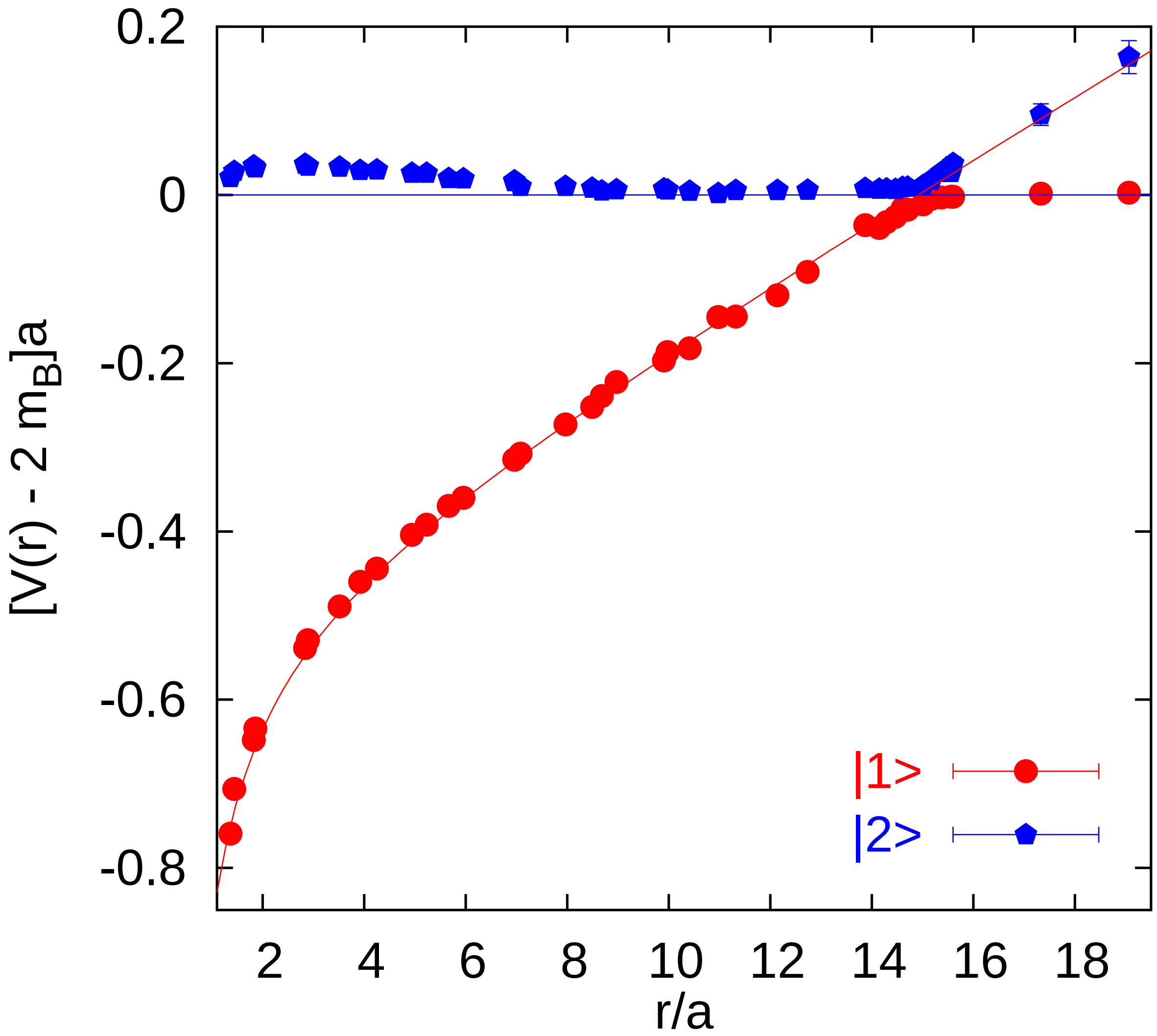
$$
\begin{aligned}
|1\rangle &= \cos\theta \, |\overline{Q}Q\rangle + \sin\theta \, |B\overline{B}\rangle \\
|2\rangle &= -\sin\theta \, |\overline{Q}Q\rangle + \cos\theta \, |B\overline{B}\rangle
\end{aligned}
$$

with $B = \overline{Q}q$.

Correlation matrix:



Static potentials GB, H. Neff, T. Düssel, T. Lippert, Z. Prkacin, K. Schilling 04-06



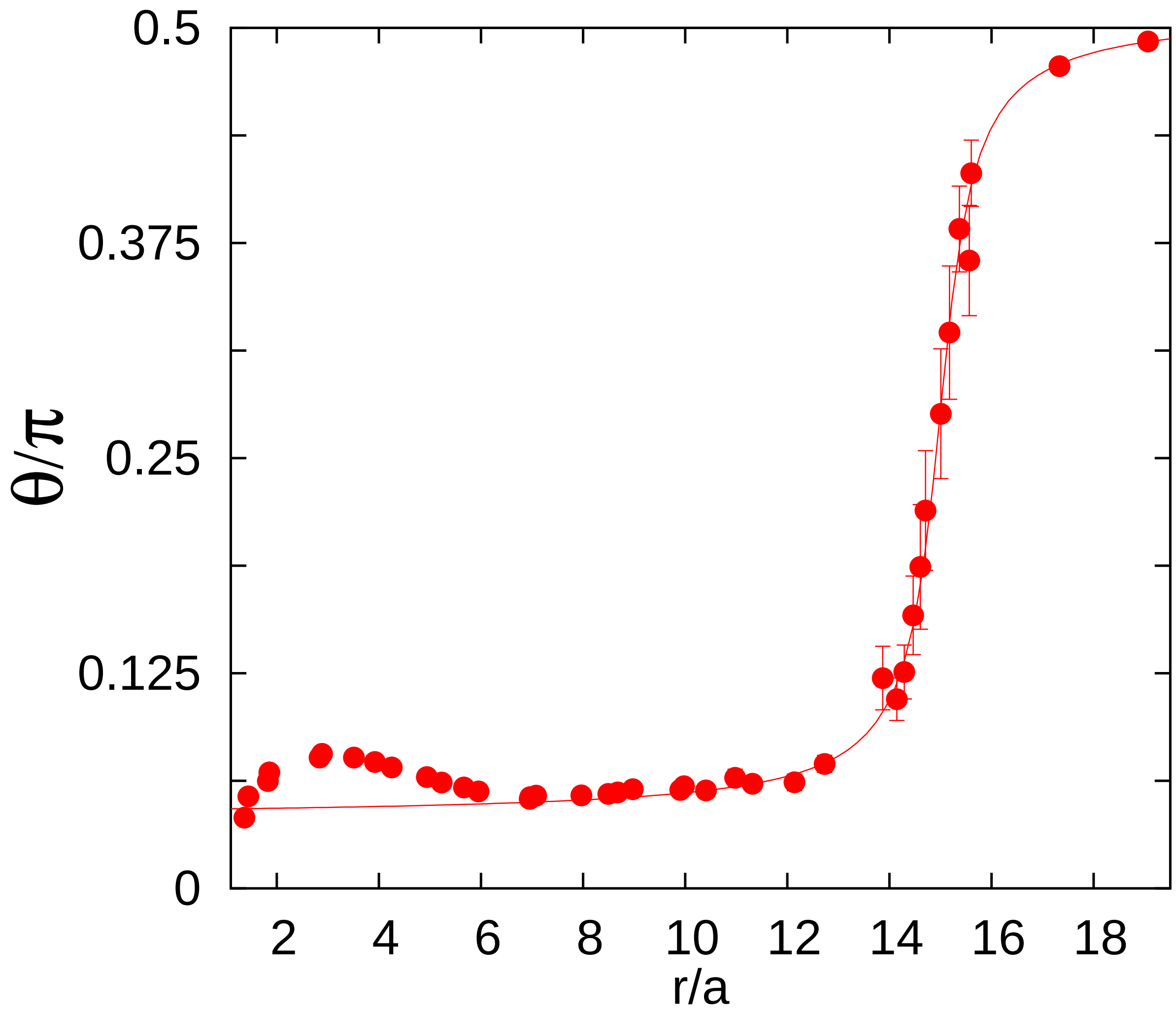
$n_f = 2$ Wilson

$m_q \approx m_s$

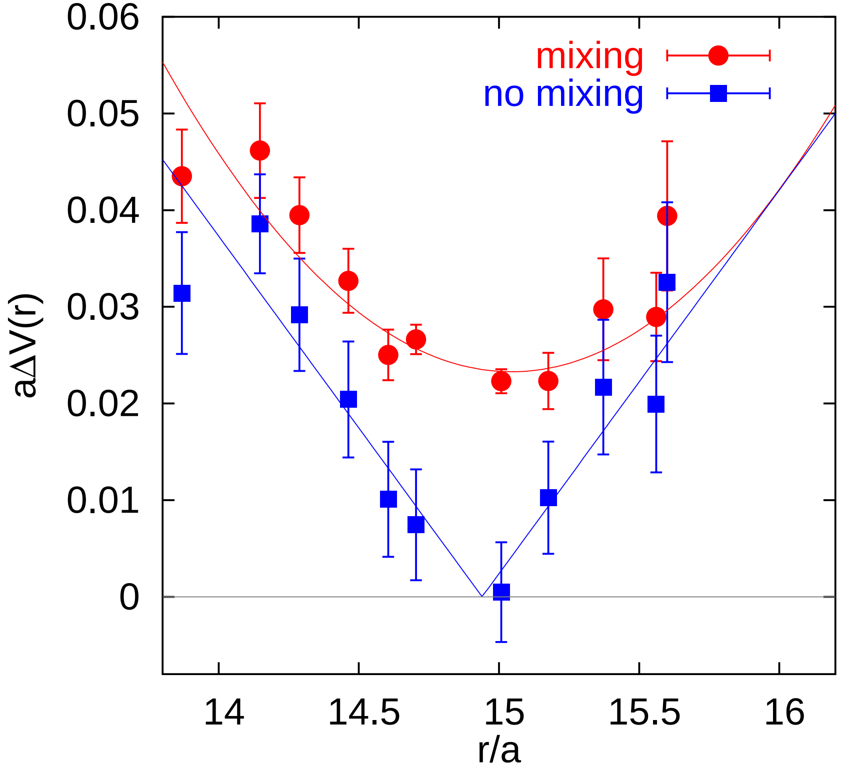
$a \approx 0.083$ fm

$a^{-1} \approx 2.37$ GeV

Mixing angle: $B\overline{B}$ content of ground state



Energy gap in the string breaking region



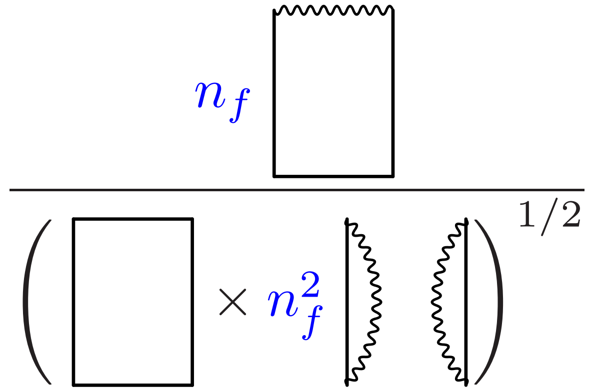
$$\Delta E(r_c) \approx 2g(r_c) \approx 51(3)\ \text{MeV}$$

Large N_c:

$SU(N_c)$ QCD $Q\overline{Q} \leftrightarrow (Q\bar{q})(q\overline{Q})$:

(also $SU(N_c)$ plus n_f fundamental Higgs)

$\Delta E_c \propto$



$\propto \sqrt{\frac{n_f}{N_c}}$

Adjoint potential $\leftrightarrow$ 2 Gluelumps:

$$\Delta E_c \propto \frac{1}{N_c}$$

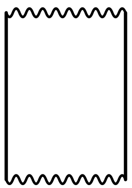
How was this possible ?

By combining different methods

- ★ Improved static action
- ★ Highly optimized test wavefunctions (smearing)
- ★ New variance reduced all-to-all quark propagator techniques

The lattice Dirac matrix $M = 1 - \kappa D$ is $12V$-dimensional.

Quarkpropagator from point x to y: M^{-1}_{yx}. Often one row is enough.

For  etc. “all-to-all” propagators are necessary:

$12V \times 12V$ elements, $V = 552960$!

Hence: Truncated eigenmode approach (TEA) with hopping parameter accelerated (HPA) stochastic estimator technique (SET). ?????????

Stochastic Estimator Techniques (SET):

N "stochastic estimates" $|\eta^i\rangle$: random vectors with components $\in \frac{\mathbb{Z}_2 + i\mathbb{Z}_2}{\sqrt{2}}$.

Notation: $\overline{O} = \frac{1}{N}\sum_{j=1}^{N} O^j \,, \quad \overline{|\eta\rangle} = \mathcal{O}(1/\sqrt{N}) \,, \quad \overline{|\eta\rangle\langle\eta|} = \mathbb{1} + \mathcal{O}(1/\sqrt{N})$

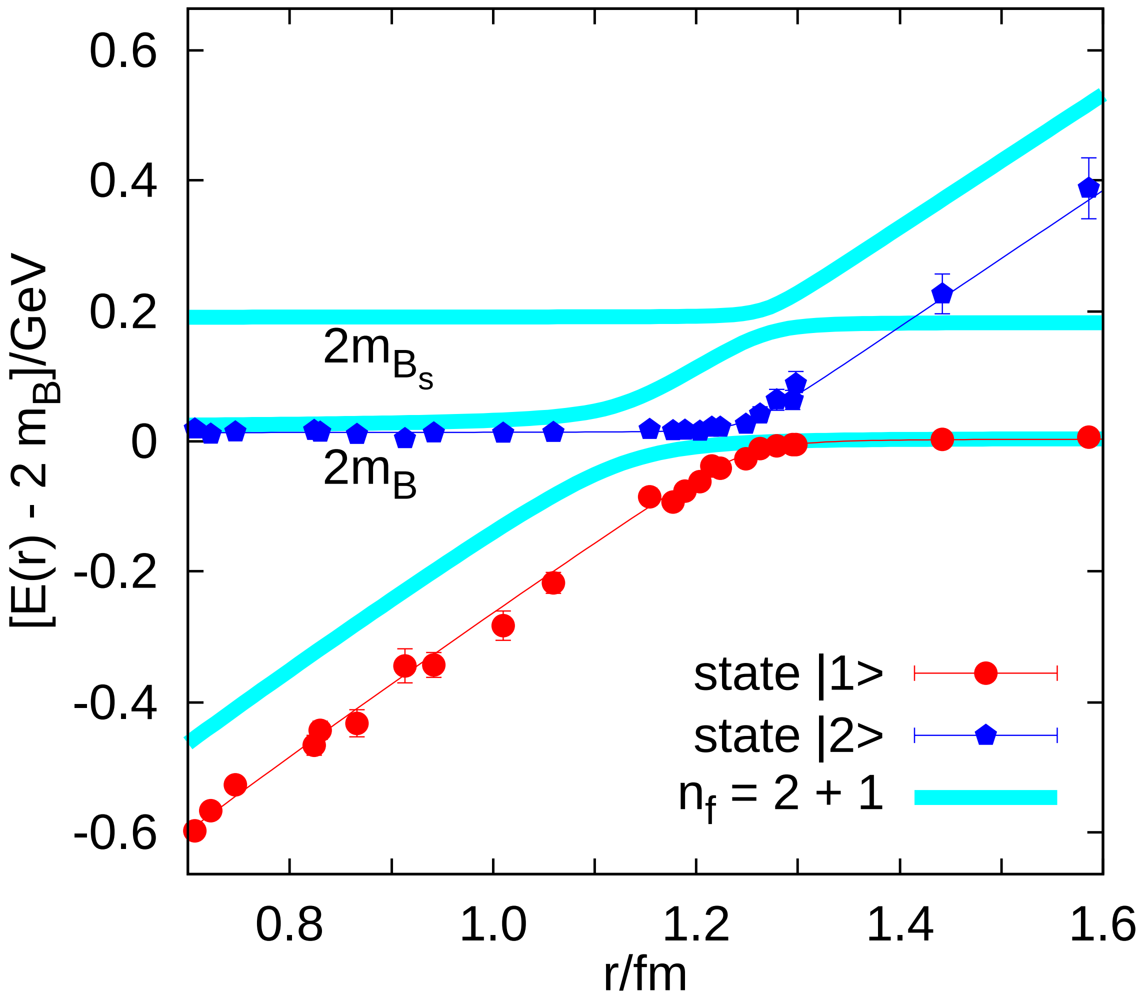
Solve linear system $M|s^i\rangle = |\eta^i\rangle$.

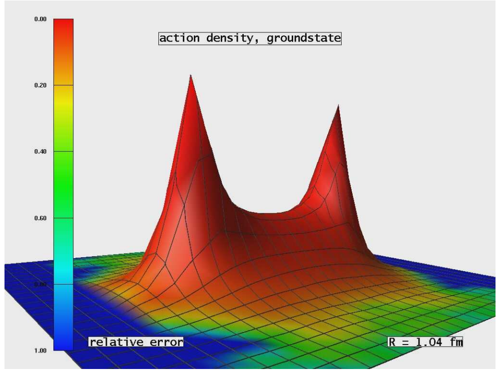
Now: $\overline{|s\rangle\langle\eta|} = M^{-1} + M^{-1}\left(\overline{|\eta\rangle\langle\eta|} - \mathbb{1}\right) = M^{-1} + \mathcal{O}(1/\sqrt{N})$.

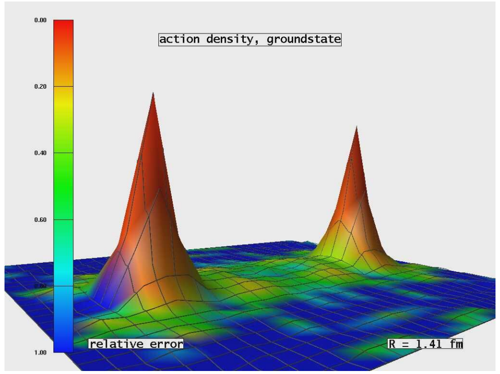
The $12V \times 12V$ problem has become a $12V \times N$ problem!

TEA and HPA used to project on space with reduced $\overline{|\eta\rangle\langle\eta|} - \mathbb{1}$.

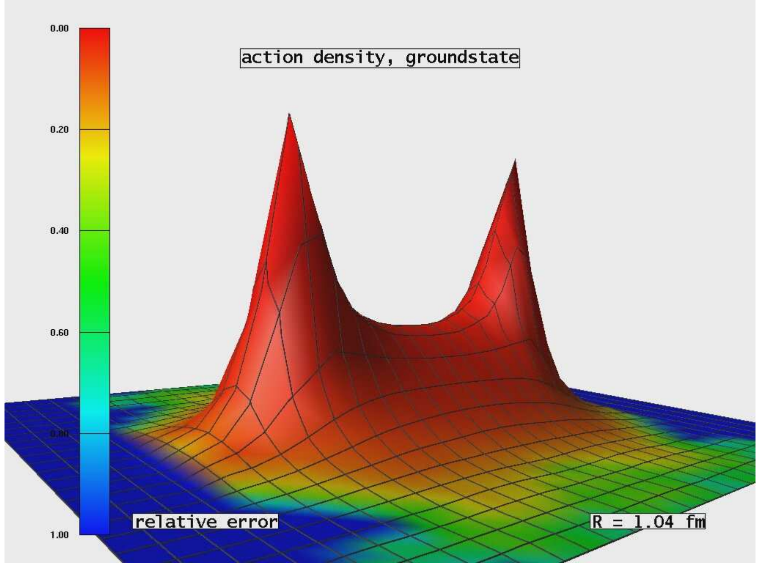
String breaking in detail

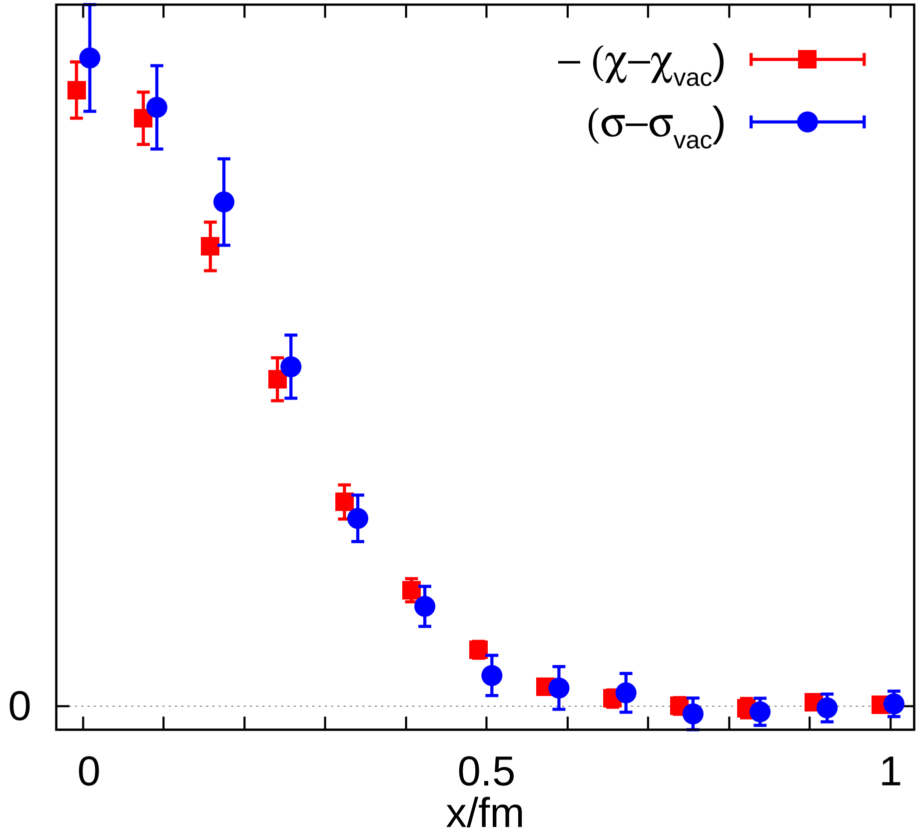






X-section at $r \approx 0.75$ fm:
chiral and action densities





Bosonic string theory

$$S_{NG} = \sigma \int d^2\zeta \, \sqrt{g} \, , \quad g^{ij} = \frac{\partial X_\mu}{\partial \zeta_i} \frac{\partial X_\mu}{\partial \zeta_j} \, , \quad i,j = 1,2 \quad \mu = 1, \ldots, d \, .$$

Choose world sheet coordinates (physical gauge):

$$s = \zeta_1 = X_{d-1} \, , \quad t = \zeta_2 = X_d \, , \quad \boldsymbol{\xi} = (X_1, \cdots, X_{d-2}) \, .$$

Expand $g = \det(g^{ij})$:

$$g = 1 + \partial_t \boldsymbol{\xi} \cdot \partial_t \boldsymbol{\xi} + \partial_s \boldsymbol{\xi} \cdot \partial_s \boldsymbol{\xi} + \cdots \, .$$

$$S = \sigma \int d^2\zeta + \frac{\sigma}{2} \int d^2\zeta \frac{\partial \boldsymbol{\xi}}{\partial \zeta_i} \cdot \frac{\partial \boldsymbol{\xi}}{\partial \zeta_i} + \cdots \, .$$

First term of an effective string action [Polchinski, Strominger 91]. One can add higher derivative operators. $\longrightarrow$ talks by Hari Dass, Matlock.

Potential between static sources [Arvis 83, Lüscher, Symanzik, Weisz 81]:

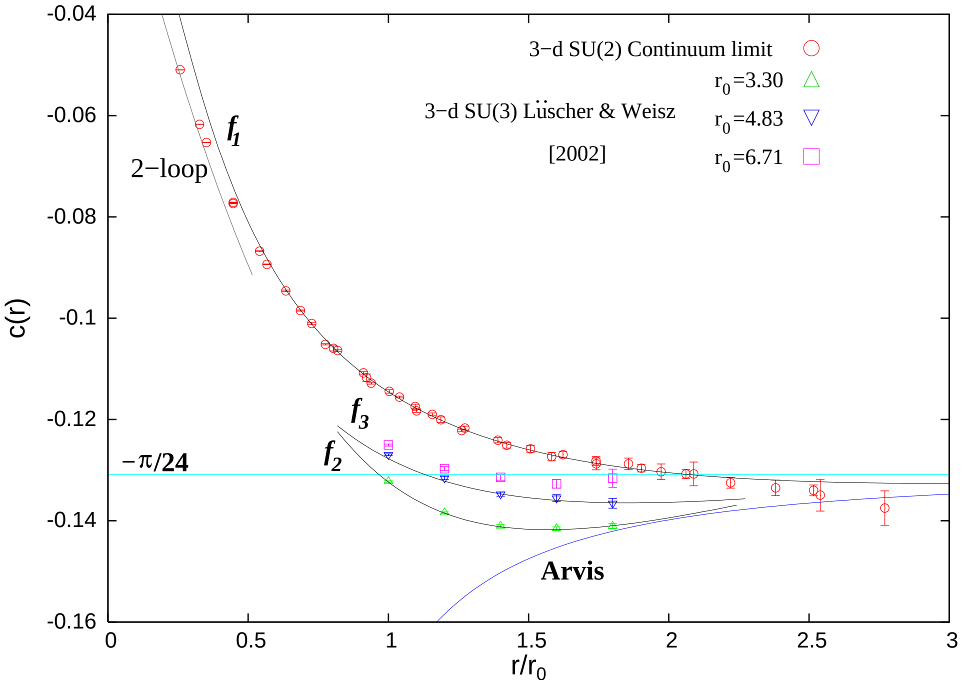
$$V(r) = c + \sigma r \sqrt{1 - \frac{(d-2)\pi}{12} \frac{1}{\sigma r^2}} = c + \sigma r - \frac{(d-2)\pi}{24} \frac{1}{r} - \frac{(d-2)^2\pi^2}{1152} \frac{1}{\sigma r^3} - \cdots$$

Lüscher, Weisz 03: no $1/r^2$ term for effective bosonic string actions.
Drummond 04, Hari Dass, Matlock: $1/r^3$ coefficient is universal too.

Hari Dass, Majumdar 07

SU(2) in $d = 2 + 1$

$r_0 \approx 0.5$ fm



open-closed string duality $\to$ mass of flux tube wrapping around a torus of circumference L:

$$m(L) = \sigma(L) L \,, \quad \sigma(L) = \sigma_0 \sqrt{1 - \frac{(d-2)\pi}{3} \frac{1}{\sigma L^2}}$$

$$\frac{\sigma}{\sigma_0}\ell = \ell - \frac{(d-2)\pi}{6\,\ell} - \frac{(d-2)^2\pi^2}{72\,\ell^3} - \frac{(d-2)^3\pi^3}{432\,\ell^5} - \cdots \,, \quad \ell = \sqrt{\sigma_0} L$$

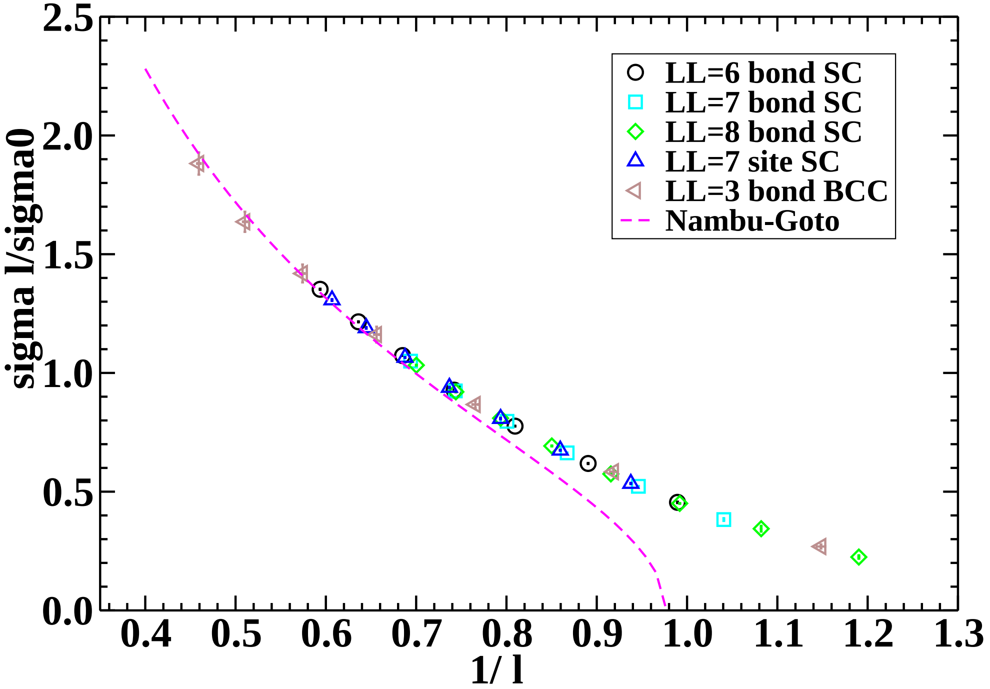
Giudice, Gliozzi, Lottini 09

$d = 3$ $\mathbb{Z}_2$ gauge theory

No sign of $1/\ell^4$.

$1/\ell^5$ coeff:

$+1/300$, not -1/432



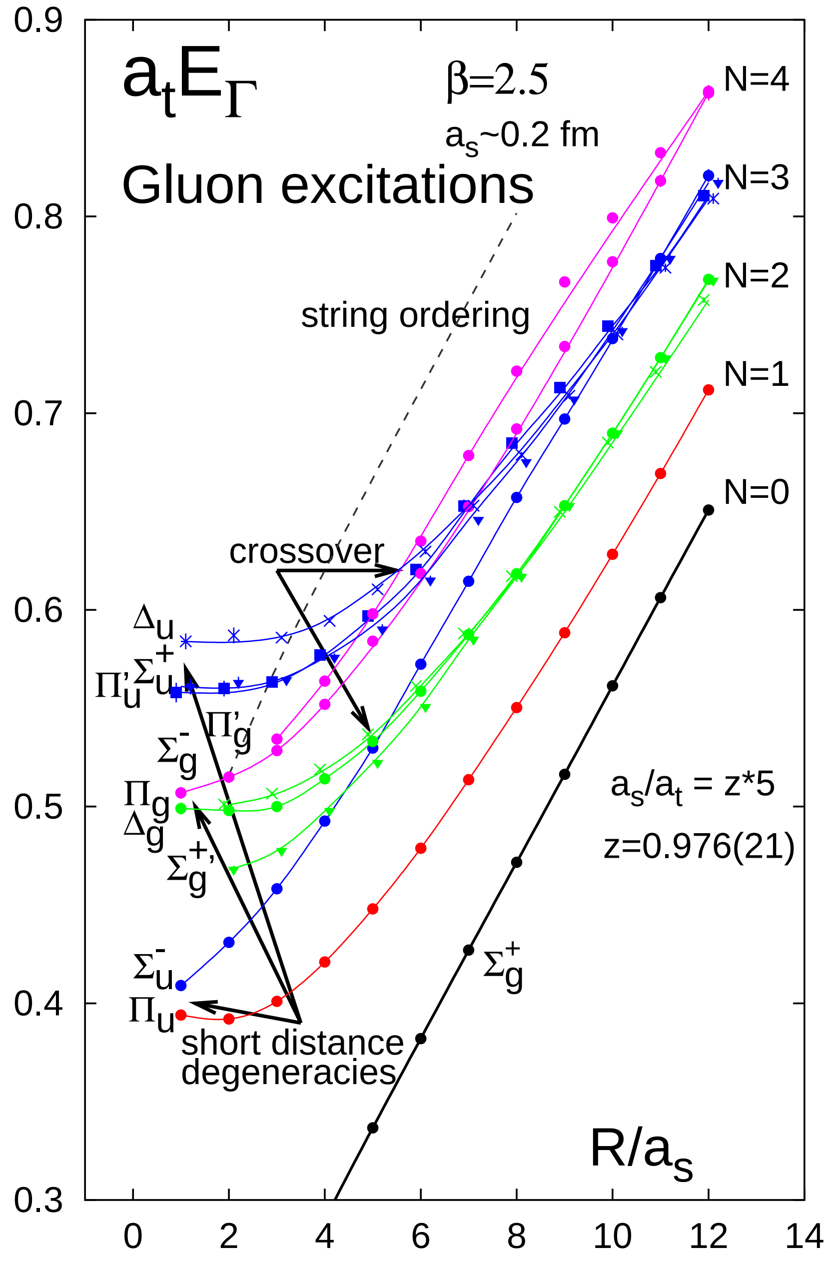
Hybrid excitations

Arvis:

$$E_n(r) = \sigma r \sqrt{1 + \left(2n - \frac{d-2}{12}\right)\frac{\pi}{\sigma r^2}}$$
$$= \sigma r - \frac{(d-2)\pi}{24r} + \frac{n\pi}{r} + \cdots .$$

Juge, Kuti, Morningstar 03

$d = 4$ $SU(3)$ gauge theory



More open strings

Energy levels in $d = 4$ SU(3) gauge theory between charges in different representation of the gauge group [Bali 00].

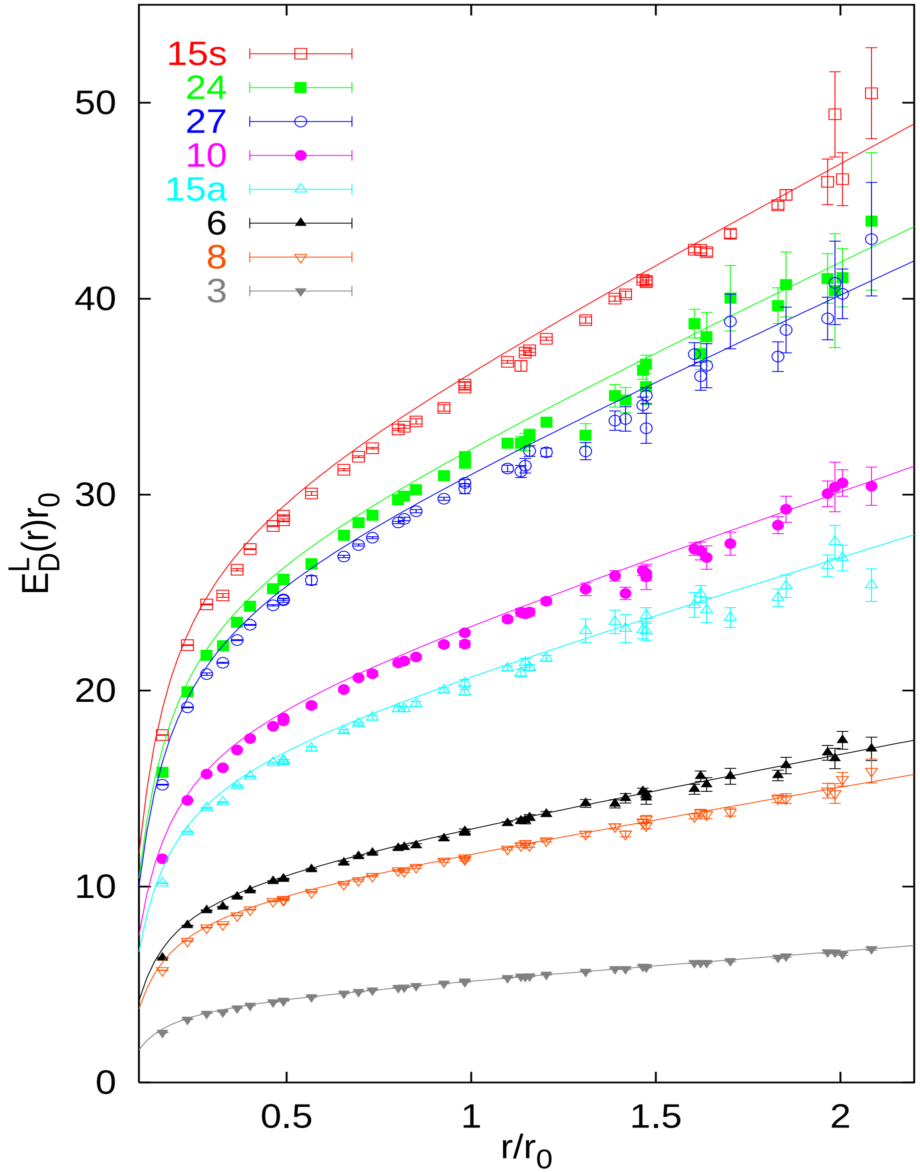
$$C_D = \frac{\text{tr}\, T_{D,a} T_{D,a}}{\text{tr}\, \mathbb{1}_D}$$

Casimir scaling:

$$V_D(r)/V_F(r) \approx C_D/C_F$$

Non-fundamental potentials have different $1/r$ coefficients.

Excitations decay for $r \to \infty$.
String tensions vanishes
$(|q-p| \pmod 3) = 0)$
or becomes the fundamental one
$(|q-p| \pmod 3) = 1, 2)$.



Closed k-strings

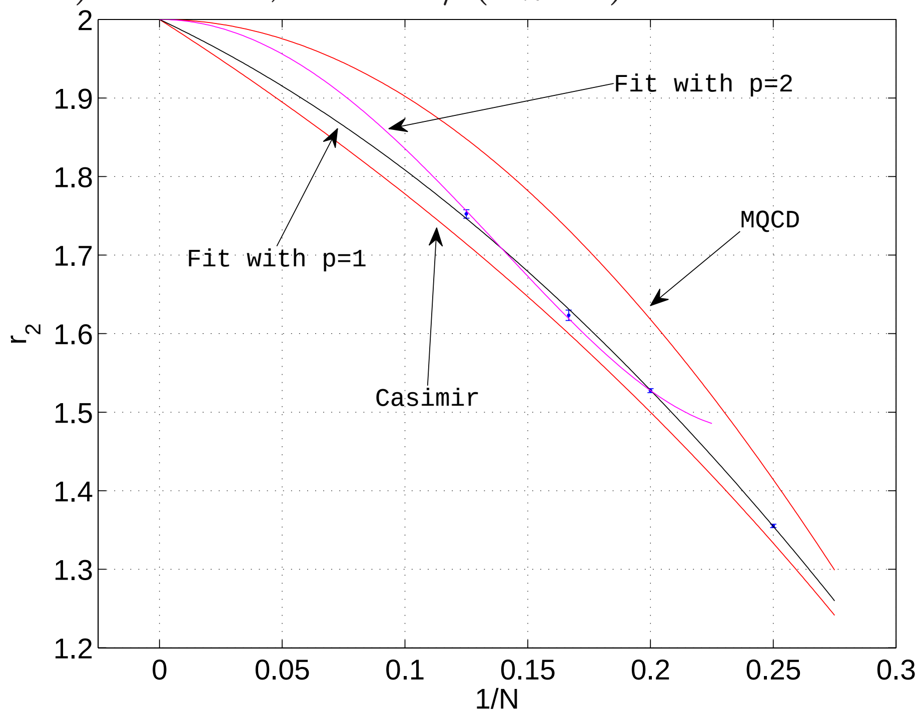
Under center transformations $z \in \mathbb{Z}_N$ a "string" in representation (p,q) of $SU(N)$ will obtain a phase $z = \exp(2\pi i(p-q)/N) = \exp(2\pi i k/N)$.
z and z^* give the same asymptotic string tension values.
Hence $k = 0, \ldots, \mathrm{int}(N/2)$ string tensions σ_k are possible ($\sigma_0 = 0$): k-strings.
Bringoltz, Teper 08: $d = 3$ $SU(4-8)$. $k = 1, 2$. $1/(\sigma_k L^3)$ term!

Casimir scaling:

$$r_k = \frac{\sigma_k}{\sigma_1} = \frac{k(N-k)}{N-1}.$$

Sine scaling [Hannani, Strassler, Zaffaroni 97, Armoni, Shifman 03]:

$$\frac{\sigma_k}{\sigma_1} = \frac{\sin(k\pi/N)}{\sin(\pi/N)}.$$



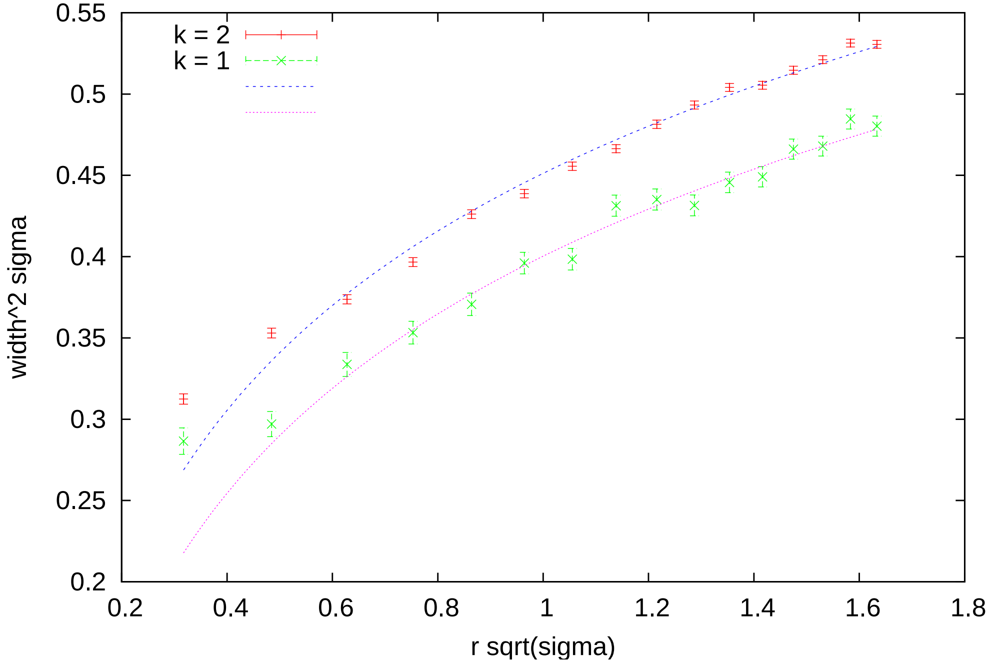
Width of (k)-strings

Quantum mechanical string fluctuations result in a flux tube with width [Lüscher, Münster, Weisz 81]:

$$\langle \varphi_k^2 \rangle = \frac{d-2}{2\pi\sigma_k} \ln \frac{r}{r_0} \,.$$

Gliozzi 06

$\mathbb{Z}_4$ gauge theory in
$d = 2 + 1$: k string widths



Baryonic potentials

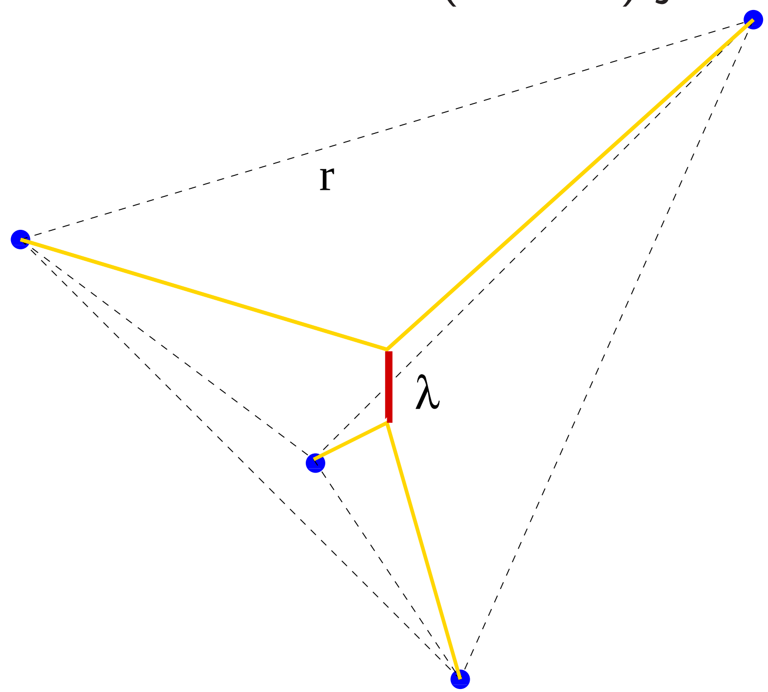
We assume $2 \leq n \leq d-1$ sources in $d-1$ spatial dimensions.

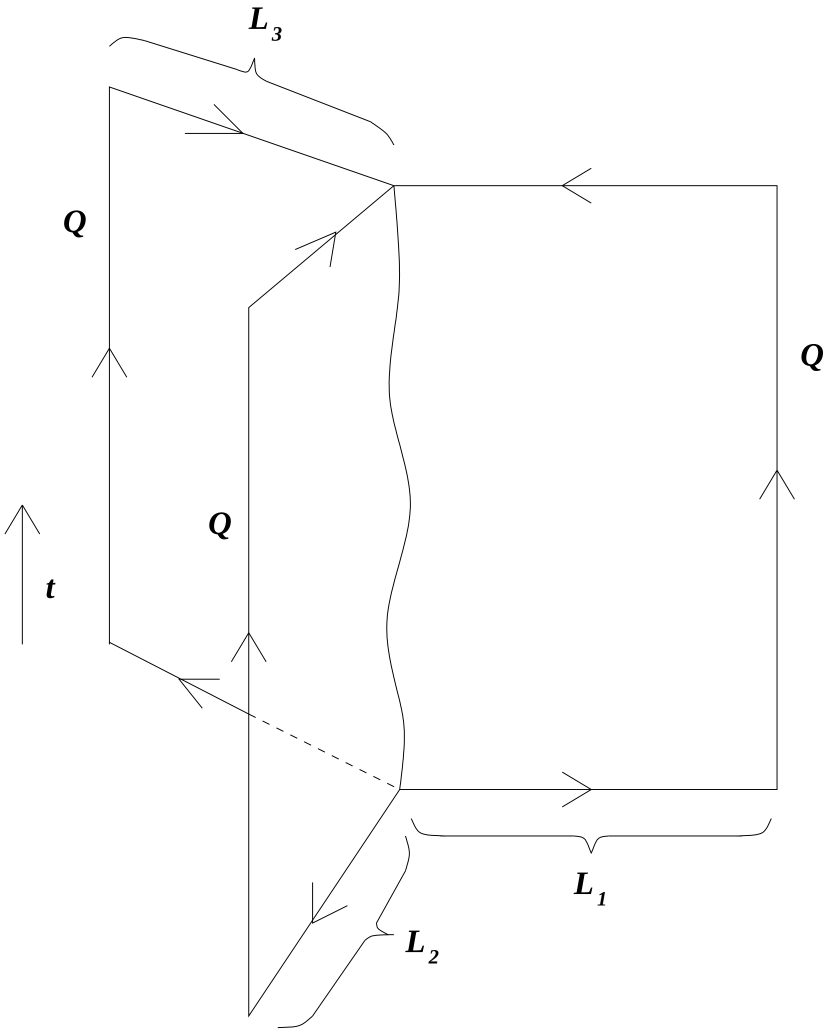
This is possible in $SU(N)$ or $\mathbb{Z}_N$ with $N = n$.

Sources define $(n-1)$ dim. hyperplane with $n-d \geq 0$ transverse dirs.

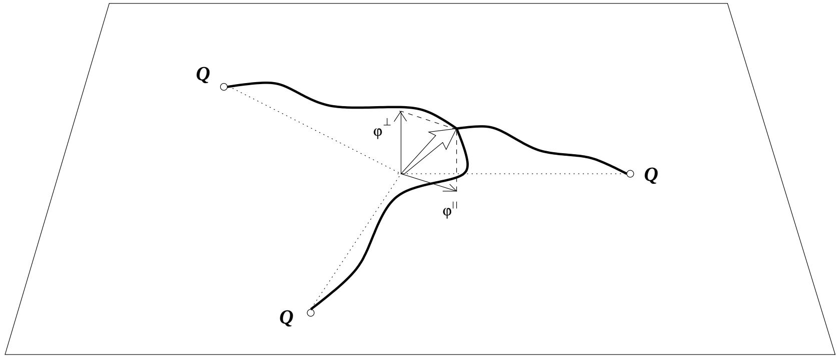
The minimal energy string configuration should have 1 (Steiner) junction!

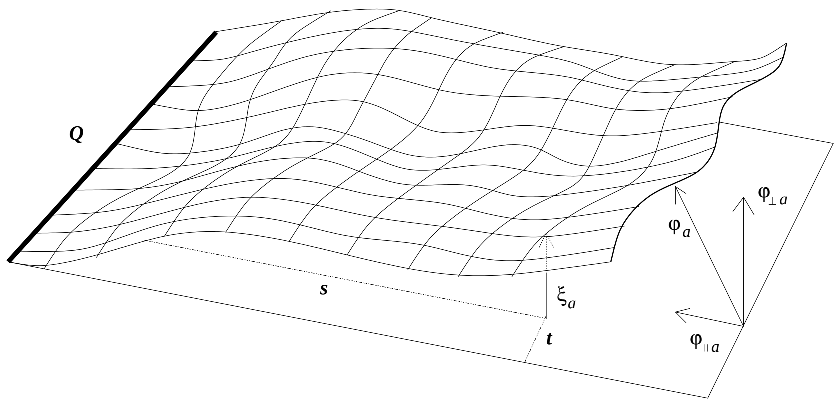
Counter example in SU(4)





$n = 3$ example: $L_Y = L_1 + L_2 + L_3$





$\boldsymbol{\xi}_a(t,s)$: worldsheet a. Junction at $s = L_a$
Equality of forces: $\sum_{a=1}^n \mathbf{e}_a = \mathbf{0}$
Junction position: $\boldsymbol{\varphi}_a(t)$

$$\boldsymbol{\xi}_a(t, L_a + \mathbf{e}_a \cdot \boldsymbol{\varphi}(t)) = \boldsymbol{\varphi}_{\perp a}(t)\,, \quad \boldsymbol{\varphi}_{\perp a} = \boldsymbol{\varphi} - \mathbf{e}_a(\mathbf{e}_a \cdot \boldsymbol{\varphi})\,.$$

NG action for these boundary conditions:

$$S = S_\| + \frac{\sigma}{2}\sum_{a,i}\int_{\Gamma_a} d^2\zeta\ \frac{\partial \boldsymbol{\xi}_a}{d\zeta_{\mathrm{i}}} \cdot \frac{\partial \boldsymbol{\xi}_a}{d\zeta_{\mathrm{i}}} + m\left(T + \frac{1}{2}\int_0^T dt\,|\dot{\boldsymbol{\varphi}}|^2\right)\,,$$

$$S_\| = \sigma \sum_a \left(L_a T + \int dt\ \mathbf{e}_a \cdot \boldsymbol{\varphi}(t)\right) = \sigma L_Y T\,.$$

Partition function:

$$Z = e^{-(\sigma L_Y + m)T}\int \mathcal{D}\boldsymbol{\varphi}\ \exp\left(-\frac{m}{2}\int dt\,|\dot{\boldsymbol{\varphi}}|^2\right)\prod_{a=1}^{3} Z_a(\boldsymbol{\varphi})\,,$$

$$Z_a(\boldsymbol{\varphi}) = \int \mathcal{D}\boldsymbol{\xi}_a \, \exp\left(-\frac{\sigma}{2}\int |\partial \boldsymbol{\xi}_a|^2\right) = e^{-\frac{\sigma}{2}\int |\partial \boldsymbol{\xi}_{\min,a}|^2} |\det(-\triangle_{\Gamma_a})|^{-\frac{d-2}{2}} \, .$$

Compactify time and introduce frequencies $w = 2\pi n/T$:

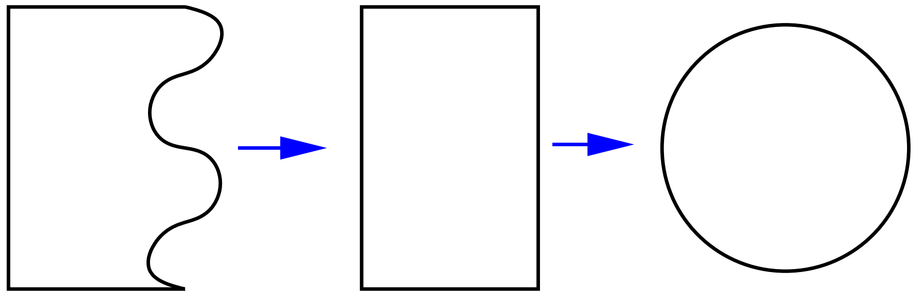
$$\boldsymbol{\xi}_{min,a} = \frac{1}{\sqrt{T}} \sum_w \boldsymbol{\varphi}_{w,\perp a} \frac{\sinh(ws)}{\sinh(wL_a)} e^{iwt} + \mathcal{O}(\varphi^2) \, ,$$

Now:

$$\int_{\Gamma_a} d^2\zeta \, \sum_i \frac{\partial \boldsymbol{\xi}_{\min,a}}{d\zeta_i} \cdot \frac{\partial \boldsymbol{\xi}_{\min,a}}{d\zeta_i} = \sum_w w \coth(wL_a) |\boldsymbol{\varphi}_{w,\perp a}|^2 + \mathcal{O}(\varphi^3) \, .$$

To calculate the Laplacian introduce conformal map on worldsheet a:

$$f_a(z) = z + \sum_w c_{wa} e^{wz} \, , \quad f_a(i\mathbb{R}) = i\mathbb{R} \, , \quad f_a(L'_a + it) = L_a + \mathbf{e}_a \cdot \boldsymbol{\varphi}(t) + it + \mathcal{O}(\varphi^2).$$



$$L'_a = L_a + \frac{1}{\sqrt{T}} \mathbf{e}_a \cdot \boldsymbol{\varphi}_0$$

$$f_a(z) = z + \frac{1}{\sqrt{T}} \sum_{w \neq 0} \frac{\mathbf{e}_a \cdot \boldsymbol{\varphi}_w}{\sinh(w L_a)} e^{wz} + \mathcal{O}(\varphi^2) \, .$$

Laplacian on worldsheet is related to Laplacian on rectangle by scale factor:

$$\triangle_{\Gamma_a} = e^{2\rho_a(z)} \triangle_{L'_a \times T} \, , \quad \rho_a(z) = -\frac{1}{2} \ln |\partial_z f_a|^2 \, .$$

$z(\tau)$: parametrization of $\partial\Gamma_a$ and $z' = dz/d\tau$. Alvarez-Polyakov:

$$\ln \frac{\det(-\triangle_\Gamma)}{\det(-\triangle_{\tilde{\Gamma}})} = \frac{1}{12\pi}\int_{\partial\Gamma} d\tau\, \frac{\epsilon_{ij} z'^i z''^j}{z'^2} \ln|\partial_z f|^2 + \frac{1}{12\pi}\int_\Gamma d^2z\, \partial_z \ln|\partial_z f|^2 \partial_{\bar{z}}\, \ln|\partial_z f|^2 .$$

First term vanishes, second term:

$$\int_{L'_a \times T} d^2 z\, \partial_z \ln|\partial_z f_a|^2\, \partial_{\bar{z}} \ln|\partial_z f_a|^2 = \sum_w w^3 |\mathbf{e}_a \cdot \boldsymbol{\varphi}_w|^2 \coth(w L_a) + \mathcal{O}(\varphi^3)\,.$$

Last step: map rectangle onto circle and use ζ-function regularisation to obtain:

$$\det(-\triangle_{L'_a \times T}) = \eta^2\!\left(\frac{iT}{2L'_a}\right) .$$

Hence:

$$\det(-\triangle_{\Gamma_a}) = \eta^2\!\left(\frac{iT}{2L'_a}\right) \exp\left(-\frac{1}{12\pi}\sum_w w^3 \coth(w L_a) |\mathbf{e}_a \cdot \varphi_w|^2\right) .$$

We can now obtain the potential up to a divergent constant from the logarithmic derivative of Z w.r.t. T, keeping only terms up to order $1/L_a$ [Jahn, de Forcrand 04, Pfeuffer, GB, Panero 08]. For $n = 3$ this reads:

$$\mathrm{V}_{qqq}(L_1, L_2, L_3) = \sigma \sum_a L_a + V^{\parallel} + (d-3)V^{\perp} + \mathcal{O}(L_a^{-2}) \,,$$

$$V^{\parallel} = -\frac{\pi}{24}\sum_a \frac{1}{L_a} + \int_0^\infty \frac{dw}{2\pi} \ln\left[\frac{1}{3}\sum_{a<b} \coth(wL_a)\coth(wL_b)\right] \,,$$

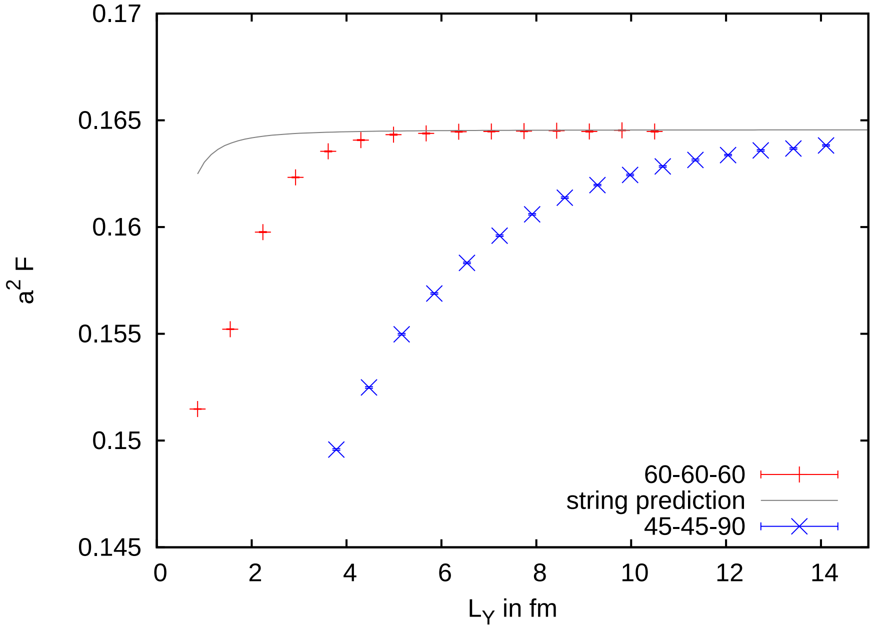
$$V^{\perp} = -\frac{\pi}{24}\sum_a \frac{1}{L_a} + \int_0^\infty \frac{dw}{2\pi} \ln\left[\frac{1}{3}\sum_{a} \coth(wL_a)\right] \,.$$

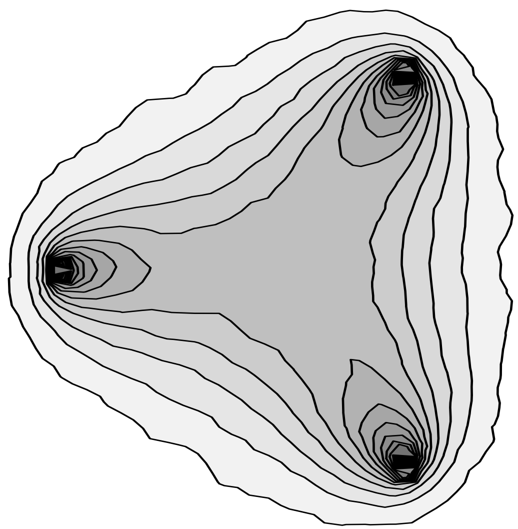
Equilateral case: $L = L_1 = L_2 = L_3$:

$$V_{qqq,\triangle}(L) = 3\sigma L - \frac{d-3}{16}\frac{\pi}{L} + \mathcal{O}(L^{-2}) \,.$$

Force in $d = 3$ $\mathbb{Z}_3$ gauge theory

Jahn, de Forcrand 05





What about the width?

Width of the junction:

$$\langle \boldsymbol{\varphi}^2 \rangle = \frac{\int \mathcal{D}\boldsymbol{\varphi}\, \boldsymbol{\varphi}^2 e^{-S}}{\int \mathcal{D}\boldsymbol{\varphi}\, e^{-S}} = \langle \boldsymbol{\varphi}^{\perp 2} \rangle + \langle \boldsymbol{\varphi}^{\parallel 2} \rangle \,.$$

We obtain [Pfeuffer, GB, Panero 08]:

$$\langle \boldsymbol{\varphi}^{\perp 2} \rangle = (d-n)\frac{1}{\pi}\int_0^\infty dw\, \frac{1}{mw^2 + \sigma w \sum_a \coth(wL_a)} \,.$$

In the equilateral case this can be be split into an integral from 0 to $w = C/L$ which is subleading in L and an integral from C/L to ∞ where we choose the constant C large enough such that $\coth(C) \approx 1$:

$$\boxed{\langle \boldsymbol{\varphi}^{\perp 2} \rangle = \frac{d-n}{n}\frac{1}{\pi\sigma}\ln\frac{L}{L_0} \,.}$$

The special case $n = d$ is included.
Also $n = 2$ reproduces the mesonic case for $r = 2L, r_0 = 2L_0$.

$\langle \boldsymbol{\varphi}^{\|2} \rangle$ diverges for $n = 2$ (expected).

For $n = 3$ the equilateral result reads:

$$\langle \varphi_{qqq}^{\|2} \rangle = \frac{4}{3} \frac{1}{\pi\sigma} \ln \frac{L}{L_c} \,.$$

This is by a factor $4/(d-3)$ larger than the perpendicular fluctuations.

$2/(d-3)$: ratio of numbers of parallel over perpendicular components.

2: ratio of perpendicular restoring force over parallel one.

Outlook

- Pure Yang-Mills theory mostly well described by bosonic string theory.
- Many universal features but where does gauge group enter?
- Regge trajectories exist in QCD but strings are broken.
- Closed k strings and (open or closed) excitations are exciting.
- Does this have to do anything with AdS/CFT?
- Could someone please simulate baryonic flux tubes?

More talks by Hari Dass, Matlock, Majumdar