Accounting for monopole configurations in Yang-Mills theory in three Euclidean dimensions

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- Quantum Chromodynamics is accepted as the theory of strong interactions.

- The ultraviolet properties are well under control thanks to renormalizability and asymptotic freedom.

- It is supposed to confine quarks in the infrared.
  Strong coupling expansion of lattice gauge theory, simulations

- **Mechanism?**-There are very few firm insights into this aspect.
  Topological configurations are believed to play a crucial role.

- **Computational scheme??**

- Georgi-Glashow model in 2+1-dimensions

- a massless ‘photon’, massive charged vector bosons and a massive Higgs

- Polyakov: non-perturbative effects change the spectrum of the theory drastically

- The action has a stable, topologically non-trivial extremum – the ’t Hooft-Polyakov monopole (playing the role of an instanton here)

- In a regime of coupling constants, a dilute gas approximation of the plasma can be justified and the effects can be reliably computed

- the ‘plasma’ of monopoles and anti-monopoles screens the photon

- The ‘dual photon’ acquires a tiny mass, $O(exp(-M_W^2/e^2))$

- confinement: ‘plasma’ of monopoles and anti-monopoles forms a dipole sheet across a Wilson loop-**area law**

Yang-Mills quantum field theory in three Euclidean dimensions

$$Z = \int D\mathbf{A} \ \exp\left(-\frac{1}{2g^2}\int d^3x \mathbf{B}^a(x)\cdot\mathbf{B}^a(x)\right) \tag{1}$$

$\mathbf{B}^a(x) = \mathbf{\nabla}\times\mathbf{A}^a(x) - \frac{1}{2}\epsilon^{abc}\mathbf{A}^b(x)\times\mathbf{A}^c(x), a = 1,2,3$

-the non-Abelian magnetic field in the vector representation of the group SO(3).

perturbation in the coupling constant g:

-super-renormalizable

-severe infrared divergences.

LGT: confining,a mass gap (and therefore no infrared divergences)

Yang-Mills potential in ’t Hooft-Polyakov ansatz for the monopole

$$A_i^a(x) = \epsilon_{iab} x^b \frac{1 - K(r)}{r^2} \qquad (2)$$

$i = 1, 2, 3$ -space index, $a, b = 1, 2, 3$ group indices. $r = \sqrt{x^a x^a}$
If $K(r) = 1 + O(r^2)$ as $r \to 0$ and $K(r) \to 0$ as $r \to \infty$: finite action
-no non-trivial classical solution with a finite action.

-rescaling: the configuration is unstable against an indefinite expansion

-telltale effects of a monopole on a large Wilson loop: contribute a phase proportional to the solid angle subtended by the loop at the monopole (center).

-We need to handle these long-range fluctuations.

-techniques required are very different from the case of the GGM.

Develop an approach which integrates perturbation theory with topological degrees:renormalization for divergences and successes of perturbation theory for ultraviolet behaviour can be preserved

Topological significance of the configurations their distinctness from perturbative fluctuations

Arafune, Freund, Goebel: topological features can be located by the zeroes of the Higgs field

For the Yang-Mills field: Abelian projection proposal of t'Hooft

Construct a composite scalar transforming in the adjoint representation of the gauge group —the fundamental Higgs of the GGM.

Defects: The locations of the zeroes depend on the composite chosen, though the net 'monopole charge' is an invariant.

Our method: R. Anishetty, P. Majumdar and H. S. Sharatchandra
E. Harikumar, I. Mitra and H. S. Sharatchandra

$S_{ij}(x) = \sum_a B_i^a(x) B_j^a(x)$

Gauge-invariant symmetric matrix

$$S_{ij}(x)\zeta_j^A(x) = \lambda^A(x)\zeta_i^A(x), \quad A = 1, 2, 3. \tag{3}$$

Monopole related to points where this matrix is triply degenerate

Moment of inertia ellipsoid

Connection with the Abelian projection proposal

$$s^{ab}(x) = \mathbf{B}^a(x)\cdot\mathbf{B}^b(x)\,, \quad (4)$$
$$s^{ab}(x)\xi_b^A(x) = \lambda^A(x)\xi_a^A(x), \quad A = 1,2,3. \quad (5)$$

Matrix s^{ab}:symmetric tensor representation of the gauge group SO(3).

Eigenvalues: same as those of S_{ij} and hence gauge-invariant,
Eigenfunctions $\xi_b^A(x)$:
- have the same topological behaviour
- adjoint representation.

One of these, say $A = 3$, plays the role of the scalar composite of t'Hooft.

An added advantage

$\xi_a^A(x)$ (after normalization): an orthonormal set and give an SO(3) matrix

Use for a local gauge transformation

Any given Yang-Mills configuration uniquely provides this gauge transformation

't Hooft-Polyakov ansatz: $(\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}, \hat{\mathbf{r}})$;
the unit vectors of the spherical coordinate system.

$$\mathbf{a}^A = \tilde{\mathbf{A}}^A + \boldsymbol{\omega}^A \tag{6}$$

$$\tilde{\mathbf{A}}^A = \xi_a^A \mathbf{A}^a \tag{7}$$

$$\boldsymbol{\omega}^A = \frac{1}{2}\epsilon^{ABC}\xi_a^B \nabla \xi_a^C \,. \tag{8}$$

$$\mathbf{a}^1 = \hat{\boldsymbol{\phi}}\frac{K(r)}{r}, \quad \mathbf{a}^2 = -\hat{\boldsymbol{\theta}}\frac{K(r)}{r}, \quad \mathbf{a}^3 = -\hat{\boldsymbol{\phi}}\frac{\cot\theta}{r}\,. \tag{9}$$

$\mathbf{a}^3$: precisely the vector potential of a Dirac monopole, with two Dirac strings along $\pm z$ directions.

Our gauge transformation, which is dictated by the gauge configuration itself, highlights the topological aspects of the configuration

$\xi_a^A(x)$ is singular along the z-axis

Non-Abelian magnetic field for the gauge potential $\boldsymbol{\omega}^A$ (which is formally a pure gauge potential) is not zero; rather,
$\mathbf{B}[\boldsymbol{\omega}] =$ Dirac string contribution.

Dirac string singularity does not contribute to the action.

$\xi_a^A(x)\xi_b^A(x) = \delta_{ab}$ everywhere.

$$\mathbf{B}^a(x)\cdot\mathbf{B}^a(x) = (\xi_a^A(x)\mathbf{B}^a(x))\cdot(\xi_b^A(x)\mathbf{B}^b(x))\,. \qquad (10)$$

$$\begin{aligned}\xi_a^A\mathbf{B}^a &= \boldsymbol{\nabla}\times(\xi_a^A\mathbf{A}^a) - \boldsymbol{\nabla}\xi_a^A\times\mathbf{A}^a - \frac{1}{2}\epsilon^{abc}\xi_a^A\mathbf{A}^b\times\mathbf{A}^c \\ &= \boldsymbol{\nabla}\times\tilde{\mathbf{A}}^A - \epsilon^{ABC}\boldsymbol{\omega}^B\times\tilde{\mathbf{A}}^C - \frac{1}{2}\epsilon^{ABC}\tilde{\mathbf{A}}^B\times\tilde{\mathbf{A}}^C \\ &= \mathbf{B}^A[\boldsymbol{\omega}+\tilde{\mathbf{A}}] - \mathbf{B}^A[\boldsymbol{\omega}] = \mathbf{B}^A[\mathbf{a}] - \mathbf{B}^A[\boldsymbol{\omega}]\,. \qquad (11)\end{aligned}$$

$\mathbf{B}^A[\mathbf{a}]$: non-Abelian magnetic field for the gauge potential $\mathbf{a}$.

$\mathbf{B}[\boldsymbol{\omega}]$: removes the Dirac string contribution from the Abelian curl of $\mathbf{a}$.

To be expected: even though our gauge transformation has singularities, it is an SO(3) matrix at each x.

Therefore, the transformed non-Abelian field strength, which was finite to begin with, remains finite everywhere.

$\boldsymbol{\nabla} \times \mathbf{a}^3$ (sans the string) is singular as $\hat{\mathbf{r}}/r^2$ at the monopole centre

This singularity is precisely cancelled by the non-Abelian interaction term $-\mathbf{a}^1 \times \mathbf{a}^2 = -\hat{\mathbf{r}}(K(r))^2/r^2$ since $K(r) = 1 + O(r^2)$ for $r \to 0$

It is not necessary to be bound to this gauge except close to the locations of monopoles (where we want all Dirac monopoles to be in the color diection $A = 3$ and all Dirac strings to be along the z-axis). We can therefore continue using the Faddeev-Popov technique (eliminating the gauge zero-modes of the fields by gauge-fixing) and keep the successes of renormalized perturbation theory.

Rewrite the Yang-Mills action in a form which explicitly highlights the monopole configurations

$\mathbf{W}^{+} = (\mathbf{a}^1 + i\mathbf{a}^2)/\sqrt{2}$ - 'charged vector boson'

$$\mathbf{a}^3 = \mathbf{A} + \mathbf{a}, \tag{12}$$

$$\mathbf{A}(x) = \sum_m q_m \hat{\boldsymbol{\phi}}_m \frac{1 - \cos\theta_m}{|\mathbf{x} - \mathbf{x_m}| \sin\theta_\mathbf{m}} \,. \tag{13}$$

$\mathbf{A}(x)$ is the superposition of Dirac potentials due to monopoles and anti-monopoles of (quantized) charges q_m located at $\mathbf{x} = \mathbf{x_m}$, $m = 1, 2, 3, \cdots$. Also, $(r_m,\ \theta_m,\ \phi_m)$ are the spherical coordinates centred at $\mathbf{x} = \mathbf{x_m}$, and $(\hat{\mathbf{r}}_m,\ \hat{\boldsymbol{\theta}}_m,\ \hat{\boldsymbol{\phi}}_m)$ are the corresponding unit vectors

$\mathbf{a}$ -' photon'

$$S = \int d^3x \left(\frac{1}{2}(-\boldsymbol{\nabla}\Phi + \nabla \times \mathbf{a} - i\mathbf{W}^{+} \times \mathbf{W}^{-})^2 + |\mathbf{D}[\mathbf{A} + \mathbf{a}] \times \mathbf{W}^{+}|^2 \right) \tag{14}$$

$$\Phi(x) = \sum_m q_m \frac{1}{|\mathbf{x} - \mathbf{x_m}|} \tag{15}$$

-(scalar) magnetic potential due to the monopoles and anti-monopoles

$$\mathbf{D}[\mathbf{A}] = \boldsymbol{\nabla} - i\mathbf{A} \tag{16}$$

-Abelian covariant derivative.

Dirac monopoles at arbitrary points with 'photons' and charged massless vector bosons $\mathrm{W}^{\pm}$ scattering off them

Even though the monopole field is singular, the action is rendered finite by the singular boundary conditions of $\mathbf{W}^{\pm}$ at the location of the monopoles.

action be finite:boundary conditions for the charged vector mesons:

$$\mathbf{W}^{+}(x) \to (\hat{\boldsymbol{\phi}}_m - i\hat{\boldsymbol{\theta}}_m)\frac{e^{i\phi_m}}{\sqrt{2}|\mathbf{x}-\mathbf{x_m}|} + O(|\mathbf{x}-\mathbf{x_m}|) \quad \text{for } \mathbf{x} \to \mathbf{x_m}. \tag{17}$$

Our ansatz and boundary condition takes into account a configuration of any number of (anti-)monopoles which have the correct form near the (anti-)monopole centers.

singular gauge:

-we have abelianized the topological aspects by having all monopoles in one colour direction

-a linear superposition of the topological configurations, to make any computation possible.

It might appear that we have expanded the action about a background (i.e. Dirac monopole) which is not a solution of the classical equations of motion. An integration over the quantum fluctuations would then wash out the effects of the background.

-the magnetic field of a Dirac monopole is a pure gradient, while that of a ‘photon’ is a pure curl.

-cross term drops out of the action -as in the case of an expansion about an extrema of the action. -the contribution of the monopole survives the integration over the gauge potential $\mathbf{a}$.

How do the topological aspects change the contribution to the partition function?

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-terms quadratic in $\mathbf{W}$ -$\mathbf{W}^{+}H\mathbf{W}^{-}$,

$$H = (\mathbf{D} \times \mathbf{D} - iq\frac{\hat{\mathbf{r}}}{r^2})\times \tag{18}$$

H : non-relativistic Hamiltonian (with unit mass) for a charged vector boson interacting with Dirac monopoles at fixed positions. -includes an anomalous magnetic moment $g = 2$ interaction (a consequence of non-Abelian gauge invariance).

H: zero modes of the form $\mathbf{D}\Lambda$ -arbitrary Λ

any gauge fixing for $\mathbf{W}$, these zero modes are absent.

we have shown: In case of one monopole, there are no other zero modes.
-(after a gauge fixing) we just get a contribution $(\mathrm{Det}H)^{-1}$.

-determinant involving the background of the Dirac monopoles depends on the scattering phase shifts and is obviously different from that of free vector bosons.

In this way the monopole configurations of the Yang-Mills theory give a contribution distinct from the perturbation theory.

Functional measure in the presence of monopole configurations

I. Mitra, HSS

Semiclassical quantization about an instanton: the measure is obtained by the collective coordinate method.

The quadratic terms in fluctuations about an instanton have zero modes related to translation and other continuous symmetries of the theory that are broken by the choice of position and other degrees of freedom of the instanton.

The fluctuations which translate the instanton (for example) are replaced by an integration over the position of the instanton using the Faddeev-Popov trick.

Even though we are not doing an expansion about a saddle point of the action, we can adopt the same strategy.

$-\boldsymbol{\nabla}\Phi + \nabla \times \mathbf{a}.$

Given a potential Φ corresponding to a configuration of monopoles and anti-monopoles of various (quantized) charges, consider another configuration where all but one (anti-)monopole, say of charge q_m and located at x_m, is displaced by an infinitesimal amount δ_j in the jth direction.

The difference in the potentials for the two configurations is precisely the potential of a dipole of moment $q_m\delta_j$.

The gradient of this dipole potential can be expressed as the curl of a vector potential.

the mode of the ‘photon’ $\mathbf{a}$ corresponding to such a dipole can be treated as a mode which displaces the monopole at x_m in the jth direction.

eliminate these modes from $\mathbf{a}$ and replace them with integration over the positions of the monopoles.

A unit factor in the partition function for each (anti-)monopole:

$$1 = \int d^3x_m \prod_{j=1,2,3} \delta(\int d^3x \partial_j \mathbf{A}(x - x_m) \cdot \mathbf{a}(x)) | \int d^3x \partial_p \partial_q \mathbf{A}(x - x_m) \cdot \mathbf{a}(x)| \quad (19)$$

∂_p :derivative with respect to the pth component of x_m and $|M_{pq}|$ stands for the determinant of a 3×3 matrix M_{pq}. The constraint is independent of the charge of the monopole.

We can use the BRST techniques to handle these constraints.

We have thus obtained integration over the positions of the monopoles from the functional measure, à la the collective co-ordinate method.

Effects of the monopole plasma from a local action

A major reason for the success in understanding the GGM is that the grand canonical partion function of the monopole plasma could be handled. We need to know the effects of an arbitrary number of monopoles and anti-monopoles at arbitrary locations on an external probe.

To do this in the present case: the first order formalism.

The first order formalism is as good as the usual second order formalism for carrying out a renormalized perturbation theory. A. Accardi, A. Belli, M. Martellini and M. Zeni

$$Z = \int D\mathbf{a} D\mathbf{W}^- D\mathbf{W}^+ D\mathbf{b} D\mathbf{b}^+ D\mathbf{b}^- \ \exp\Big(\int d^3x (- g^2 (\frac{1}{2}\mathbf{b}^2 + \mathbf{b}^+ \cdot \mathbf{b}^-) + i\mathbf{b} \cdot (-\boldsymbol{\nabla}\Phi$$

$$+\boldsymbol{\nabla} \times \mathbf{a} + i\mathbf{W}^+ \times \mathbf{W}^-) + i\mathbf{b}^+ \cdot \mathbf{D}[\mathbf{A} + \mathbf{a}] \times \mathbf{W}^- + i\mathbf{b}^- \cdot \mathbf{D}[\mathbf{A} + \mathbf{a}] \times \mathbf{W}^+)\Big)$$

(+ gauge fixing +the ghost terms)

Gradient and curl parts of $\mathbf{b}$:

$$\mathbf{b} = \boldsymbol{\nabla}\chi + \boldsymbol{\nabla} \times \mathbf{c} \quad (20)$$

Part of the partition function involving $\mathbf{b}$ takes the form,

$$
\begin{aligned}
Z \;=\; & \int D\mathbf{a}D\mathbf{W}^{-}D\mathbf{W}^{+}D\mathbf{b}^{+}D\mathbf{b}^{-}D\chi D\mathbf{c}\ \text{exp}\Big(\int d^3x\big(-\frac{g^2}{2}((\boldsymbol{\nabla}\chi)^2+(\boldsymbol{\nabla}\times\mathbf{c})^2) \\
& +\chi(x)\boldsymbol{\nabla}\cdot(\mathbf{W}^{+}\times\mathbf{W}^{-})+i(\boldsymbol{\nabla}\times\mathbf{c})\cdot(\boldsymbol{\nabla}\times\mathbf{a}+i\mathbf{W}^{+}\times\mathbf{W}^{-})\big)-i\sum_m q_m\chi(x_m) \\
& + \text{other terms}\Big)\,.
\end{aligned}
$$

Here we have used

$$\boldsymbol{\nabla}^2\Phi(x) = -\sum_m q_m\delta(x-x_m)\,. \qquad (21)$$

χ is the ‘dual photon’

The ‘photon’ field $\mathbf{a}$ is useful for renormalized perturbation theory, while the ‘dual photon’ χ incorporates the topological degrees of freedom.

Both the photon and the dual photon are simultaneously present in our formulation.

It is thus close in spirit to the two-photon formulation of Zwangiger

Free Maxwell theory in three Euclidean dimensions:

$$Z = \int D\mathbf{a} \ \exp\left(-\frac{1}{2g^2}\int d^3x (\boldsymbol{\nabla} \times \mathbf{a}(x))^2\right) . \tag{22}$$

-auxiliary field $\mathbf{b}$

$$Z = \int D\mathbf{b} D\mathbf{a} \ \exp\left(\int d^3x (-\frac{g^2}{2}\mathbf{b}(x)^2 + i\mathbf{b}(x) \cdot \nabla \times \mathbf{a}(x))\right) . \tag{23}$$

-note $i = \sqrt{-1}$

$$Z = \int D\mathbf{b} \ \prod_x \delta(\nabla \times \mathbf{b}(x)) \exp\left(-\frac{g^2}{2}\int d^3x \mathbf{b}(x)^2\right) . \tag{24}$$

$\mathbf{b}(x) = \boldsymbol{\nabla}\chi(x)$.

$$Z = \int D\chi \ \exp\left(-\frac{g^2}{2}\int d^3x (\boldsymbol{\nabla}\chi(x))^2\right) \tag{25}$$

only one transverse degree of freedom in three dimensions:scalar χ

Local field theory of interactions of monopole charges and electric currents in three Euclidean dimensions

Sources j of Dirac monopoles interacting with electric currents $\mathbf{J}$.

$$
\begin{aligned}
Z &= \exp\Big(\int d^3x d^3y(-\frac{1}{2g^2}j(x)D(x-y)j(y) + i\,j(x)A_i(x-y)J_i(y) \\
&\quad -\frac{g^2}{2}J_i(x)D_{ij}(x-y)J_j(y))\Big)\,. \qquad (26)
\end{aligned}
$$

$D(x-y)$ and $D_{ij}(x-y)$ -free propagators for massless scalar and massless vector fields in coordinate space.

$A_i(x-y)$ -Dirac vector potential of a monopole

-here playing the role of the propagator connecting monopole charges to electric currents.

-$\sqrt{-1}$ in the interaction of magnetic monopoles and electric currents coupling constants appearing in inverse proportions

The Dirac potential at x due to a monopole at x': the Green function for the operator $\partial_3 \boldsymbol{\nabla}^2$:van Baal

$$\begin{aligned} \mathbf{A}(x-x') &= \hat{\mathbf{n}}_3 \times \boldsymbol{\nabla} \ln(|x-x'| + x_3 - x'_3) \quad (27) \\ &= -4\pi \hat{\mathbf{n}}_3 \times \boldsymbol{\nabla} (\partial_3 \nabla^2)^{-1} (x-x')\,. \quad (28) \end{aligned}$$

-can be written as a local quantum field theory involving monopole charges, photons and electric currents:

$$\begin{aligned} Z &= \int D\mathbf{a} D\chi \; \exp\Big(\int d^3x \big(-\frac{1}{2}(\partial_3 \chi(x))^2 - \frac{1}{2}(\hat{\mathbf{n}}_3 \times (\boldsymbol{\nabla} \times \mathbf{a}(x)))^2 \\ & \quad + i \partial_3 \chi(x) \hat{\mathbf{n}}_3 \cdot \boldsymbol{\nabla} \times \mathbf{a}(x) + i g J_i(x) a_i(x) + i g^{-1} j(x) \chi(x) \big) \Big) \quad (29) \end{aligned}$$

-functional integration over $\mathbf{a}(k)$ and $\chi(k)$, recovers quadratic interactions between the currents.

-'photon' quanta described simultaneously by the dual fields, the usual vector potential $\mathbf{a}$ and the dual scalar χ.

-the analogue of the two-potential formalism of Zwanziger for quantum electrodynamics of monopoles and charges in three Euclidean dimensions.

-characteristic appearance of $\hat{\mathbf{n}}_3 \cdot \boldsymbol{\nabla} \times \mathbf{A}$ in the terms involving the photon and the dual photon.

-only the ∂_3 derivative of χ

-χ couples to the monopoles locally as the dual photon potential should.

-If we consider only monopoles and anti-monopoles of unit charge summing over these charges we get $\cos\chi(x)$. A sum over arbitrary number of monopoles and anti-monopoles exponentiates this into a new term in the action. This gives a mass to the 'dual photon' and results in area law for the Wilson loop.

Situation is more complicated in our case. We also have to handle the interactions of the monopoles with the massless charged vector bosons.

Describe the net effect of the monopole plasma by a local field theory.

The interaction of a monopole at x_m with an electric current at x can be put in the convenient form

$$i\int d^3x \mathbf{A}(x-x_m)\cdot\mathbf{J}(x) = i\,4\pi\int d^3x(\partial_3\nabla^2)^{-1}(x-x_m)\hat{\mathbf{n}}_3\cdot\boldsymbol{\nabla}\times\mathbf{J}(x) \quad (30)$$

-sum over m corresponding to monopole charges q_m in the present case.

Using auxiliary scalars ϕ and ψ: local form

$$\int D\phi D\psi\ \exp\left(i\int d^3x\psi(x)\partial_3\nabla^2\phi(x) + i\sum_m q_m\phi(x_m) + i\,4\pi\int d^3x\psi(x)\hat{\mathbf{n}}_3\cdot\boldsymbol{\nabla}\times\mathbf{J}(x)\right)$$

-the scalar ϕ couples locally to the monopoles just as χ

Summing over $q_m = \pm 1$ we get $\cos(\phi(x) - \chi(x))$ in the action.

The combination $\phi(x) - \chi(x)$ plays the role of $\phi(x)$ of the GGM.

Due to the collective coordinates non-local constraints on the 'photon'.

-make them local: introducing an auxiliary scalar $\eta(x)$:

$$\sum_{N=0}^{\infty} \frac{1}{N!} \int D\eta \prod_x \delta(\partial_3 \boldsymbol{\nabla}^2 \eta(x) + 4\pi \hat{\mathbf{n}}_3 \cdot \boldsymbol{\nabla} \times \mathbf{a}(x)) \times \prod_{m=1}^{N} \int d^3 x_m \prod_{j=1,2,3} \delta(\partial_j \eta(x_m)) |\partial_p \partial_q \eta(x_m)| \,. \quad (31)$$

-the auxiliary scalar η has an extremum at the locations of the (anti-)monopoles

$|\partial_p \partial_q \eta|$ is the determinant of the quadratic fluctuations at these extrema.

-Faddeev-Popov procedure.

-for the determinant we can use ghost fields.

The generic configuration has half-monopoles joined by Z_2 strings.

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The triple degeneracy is the extreme limit where the two half monopoles are collapsed on each other. In this paper we restrict our concern to the point singularities i.e. the (anti-)monopoles. It is possible that very long Z_2 strings are the dominant configurations. This more general case can also be handled by our techniques.

Summary

To handle confinement we need techniques to sum over topologically non-trivial configurations beyond a semi-classical approximation

-describe topology locally

-use this to get a singular gauge transformation which highlights the topological objects

-incorporate the effects of the topological configurations in auxiliary fields of the first order formalism; this way the renormalized perturbation theory is still possible

-our description using singular configurations highlights how the topological aspects give contributions different from perturbation theory

-it also helps in obtaining the functional measure

-we can sum over positions of the topological objects in favour of a local action

-effects are contained in new terms involving the auxiliary fields of the first order formalism

-new Feynman rules agreeing with the usual at large momenta: renormalization of uv divergences

-infrared behaviour controlled

-3+1-d: some additional techniques: HSS

-d-branes in field theory?