Nucleon properties in QCD sum rules

The nucleon mass Ioffe 1981. also delta mass
Resonance masses N* , higher spin in I=1/2 ,3/2
The magnetic moment, static , transition moments
Axial coupling , Bjorken sum rule

$$\langle p,s|\,\frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u+\frac{1}{9}\bar{d}\gamma_\mu\gamma_5 d+\frac{1}{9}\bar{s}\gamma_\mu\gamma_5 s\,|p,s\rangle=-s_\mu G_{Bj} \tag{1}$$

- Sigma term , n-p mass difference and other multiplets splittings
- In nuclear medium properties , scalar and vector potentials , quenching ,Okhamoto- Nolan Schrieffer anomaly
- And many others.
- Sketch Bjorken matrix element , with QSR. The calculation is related to the derivative of topological susceptibility, and eta , eta prime couplings to octet and single currents

- QCD sum rule results have errors which are difficult to estimate.
- One can expect 10 to 20 per cent errors , some cases one does better
- There is at least , where simple calculation fails – sigma term
- Value of the method , its relative simplicity in calculating a large variety of physical quantities .
- Especially useful is the corresponding physical quantities are calculates by other methods – a few examples , derivative of the topological susceptibility, mass of etaprime in the chiral limit , matrix of eta and etaprime to octet and singlet currents.
- Different view , like for example pattern of baryon magnetic moments

Let us briefly consider the method of QCD sum rules. Denoting generically

$$F(q^2) = \frac{i}{\pi}\int d^4x e^{iqx} \langle 0 | T\{A(x), B(0)\} | 0 \rangle$$

where A(x) and B(x) are the local fields that connect the vacuum to the hadronic state of interest, one considers the dispersion relation

$$F(q^2) = \frac{1}{\pi}\int \frac{\text{Im}\, F(s)}{s-q^2} ds + \text{subtractions}$$

and Borel transforms it to obtain

$$\hat{B}\, F(q^2) = \frac{1}{\pi}\int \text{Im}\, F(s) e^{-s/M^2} ds \qquad \ldots(27)$$

where the Borel transform is defined by

$$\hat{B}\, F(q^2) = \lim_{-q^2\to\infty, n\to\infty} \left[\frac{(-q^2)^{n+1}}{n!} \left(\frac{d}{dq^2} \right)^n F(q^2) \right]_{-q^2/n=M^2}$$

Now the left hand side of Eq.(27) is computed using the operator product expansion while the right hand side is written in the form

$$\frac{1}{\pi}\int \mathrm{Im}\,F(s)e^{-s/M^2}\,ds = \lambda_{\mathrm{H}}\; e^{-m_H^2/M^2} + \frac{1}{\pi}\int_{W^2}^{\infty} \mathrm{Im}\,F(s)e^{-s/M^2}\,ds \qquad \text{… (27a)}$$

where λ_{H} is the coupling involving the lowest mass state H in the dispersion representation:

$\mathrm{Im}\,\mathrm{F(s)} = \pi\,\lambda_{\mathrm{H}}\;\delta\left(s-m_H^2\right)$ + contributions from higher mass states.

This leads to

$$\lambda_{\mathrm{H}}\; e^{-m_H^2/M^2} \;=\; \hat{B}\;\mathrm{F(q^2)} - \frac{1}{\pi}\int_{W^2}^{\infty} \mathrm{Im}\,F(s)e^{-s/M^2}\,ds\,. \qquad \text{….(27b)}$$

$$\langle 0|\,T(q_i^a,\bar{q}_k^b)\,|0\rangle=\frac{i\delta^{ab}}{2\pi^2}\frac{(\hat{x})_{ik}}{x^4}-\frac{\delta^{ab}}{2\pi^2}g_q\frac{x\cdot Z(\hat{x})_{ik}}{x^4}\gamma_5+\frac{i}{32\pi^2}g_c\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\frac{(\hat{x}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{x})_{ik}}{x^2}$$

$$-\tfrac{1}{12}\delta^{ab}\delta_{ik}\langle 0|\,\bar{q}q\,|0\rangle+\tfrac{1}{12}g_q\chi\delta^{ab}(\hat{Z}\gamma_5)_{ik}\langle 0|\,\bar{q}q\,|0\rangle+\tfrac{1}{36}g_q x^\alpha Z^\beta\sigma_{\alpha\beta}\gamma_5\langle 0|\,\bar{q}q\,|0\rangle$$

$$+\frac{\delta^{ab}}{192}\delta_{ik}x^2\langle 0|\,\bar{q}\sigma\cdot Gqg_c\,|0\rangle+\frac{\delta^{ab}g_q\kappa\langle\bar{q}q\rangle}{72}[(\tfrac{5}{2}x^2\hat{Z}-x\cdot Z\,\hat{x})\gamma_5]_{ik}+\text{higher-order terms}\ .$$

$$a=-(2\pi)^2\langle 0|\,\bar{q}q\,|0\rangle\ ,$$

$$b=\langle 0|\,g_c^2G^n_{\mu\nu}G^{n\mu\nu}\,|0\rangle\ ,$$

$$\alpha_s=g_c{}^2/4\pi\ ,$$

$$L=\ln(M^2/\Lambda^2)/\ln(\mu^2/\Lambda^2)\ ,$$

$$am_0{}^2=(2\pi)^2\langle 0|\,\bar{q}\sigma\cdot Gq\,|0\rangle\ ,$$

The experimental value[6] of the proton magnetic moment is 2.793 nuclear magnetons and has at first sight no natural explanation. We shall show that from the point of view of QCD it is natural to regard this number as

$$\mu_p = \tfrac{8}{3}(1+\delta_p)\frac{e\hbar}{2cM_N} , \tag{1.1}$$

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where $\delta_p = 0.0473$ is a small calculable correction. Similarly, the hyperon magnetic moments, whose experimental values appear to be a melange of arbitrary numbers, assume a neat pattern when viewed in a similar fashion. For example, writing

Baryon	$e\hbar/2M_Nc$	$e\hbar/2M_Bc$	$(1+\delta_B)$
p	$+2.793$	$+2.793$	1.047
n	-1.913	-1.913	1.435
Σ^+	$+2.379$	$+3.012$	1.129
Σ^-	-1.12	-1.42	1.063
Ξ^-	-0.69	-0.97	0.73
Ξ^0	-1.25	-1.76	1.32

$$\mu_\Lambda = \tfrac{2}{3}(e_u + e_d + 4e_s)\frac{e\hbar}{2M_\Lambda c}(1+\delta_\Lambda) \tag{2}$$

and compute δ_Λ. Using the experimental values of μ_Λ and M_Λ Eq. (2) yields

$$\delta_\Lambda = 0.093 \ . \tag{3}$$

and the susceptibilities are defined by

$$\langle \bar{q}\sigma_{\mu\nu}q \rangle_F \equiv \chi e_q F_{\mu\nu} \langle \bar{q}q \rangle ,$$

$$\langle \bar{q}g_s G_{\mu\nu}q \rangle_F \equiv \kappa e_q F_{\mu\nu} \langle \bar{q}q \rangle ,$$

$$\epsilon_{\mu\nu\rho\sigma} \langle \bar{q}g_s G^{\rho\sigma}\gamma_5 q \rangle_F \equiv i\xi e_g F_{\mu\nu} \langle \bar{q}q \rangle ,$$

giving good agreement were found at $\chi = -3$ GeV^{-2} and $2\kappa - \xi = 3$ with $\kappa = 0.75$, where the value of χ is close to that of the one-pole model.[1] These results together with the experimental values are given in Table I.

TABLE I. Comparison between experimental values of baryon-octet magnetic moments and the theoretical values obtained by the ratio method. Magnetic moments are given in units of the proton magneton.

Baryon	Experiment	Ratio method
p	$+2.793$	$+3.04$
n	-1.913	-1.79
Σ^+	$+2.379$	$+2.73$
Σ^-	-1.12	-1.26
Ξ^-	-0.69	-0.93
Ξ^0	-1.25	-1.32
Λ	-0.61	-0.50

$$(M_n - M_p)_{\mathrm{elec}} = -0.76 \pm 0.30 \text{ MeV} .$$

The experimental mass difference is 1.29 MeV,

Consider the quark mass term in the QCD Hamiltonian density $\mathcal{H}_{\mathrm{QCD}}$, as given by

$$\begin{aligned}\mathcal{H}_{\mathrm{mass}} &= m_u \overline{u}u + m_d \overline{d}d + m_s \overline{s}s + \cdots \\ &= \hat{m}(\overline{u}u + \overline{d}d) - \frac{1}{2}\delta m(\overline{u}u - \overline{d}d) + m_s \overline{s}s + \cdots .\end{aligned}$$

$$\delta(M_h) = \frac{-\delta m}{2} \frac{\langle h|(\overline{u}u - \overline{d}d)|h\rangle}{2M_h} .$$

$$(M_n - M_p)_q = \delta m \frac{\langle p|\overline{u}u - \overline{d}d|p\rangle}{2M_p} .$$

$$H \equiv \frac{\langle p|\overline{u}u - \overline{d}d|p\rangle}{2M_p} ,$$

$$(M_n - M_p)^{\mathrm{expt}} = -0.76 + \delta m H .$$

$$(M_n - M_p)_q^{\mathrm{expt}} = 2.05 \pm 0.30 \text{ MeV} .$$

$$1.75 \text{ MeV} \le (M_n - M_p)_q \le 2.35 \text{ MeV} ,$$

As a final remark we shall comment on the Nolen-Schiffer anomaly [33]. We saw in the last section that the empirical neutron-proton mass difference can be written as

$$(M_n - M_p)^{\mathrm{expt}} = -0.76 + \delta m H \ . \qquad (4.1)$$

Now, it is well known that the matrix element of the axial vector current is quenched [34] inside the nuclear medium by about 30%. It may be reasonable to assume that the isovector-scalar matrix element $\langle p|\overline{u}u - \overline{d}d|p\rangle/2M_p$ is also quenched in a similar fashion.

Assuming then, for example, a 30% reduction in the value of H, it follows from Eq. (4.1) that the effective mass difference in the nuclear medium is $(M_n - M_p)^{\mathrm{expt}}_{\mathrm{med}} \simeq 0.49$ MeV. Understanding the Nolen-Schiffer anomaly is then reduced to explain the quenching of $\langle p|\overline{u}u - \overline{d}d|p\rangle/2M_p$ in a nuclear medium. This can be handled either by traditional nuclear structure calculations or again by use of QCD sum rules as in the quenching of nucleon axial vector coupling [35,36].

The QCD sum-rule method is basically the following. To compute the properties of a given hadron, one chooses a current which has a nonzero matrix element between the physical vacuum and the hadron in question. For the case of the proton we shall use the current[2,7]

$$\eta(x)=[u^a(x)C\gamma_\mu u^b(x)]\gamma_\mu\gamma^5 d^c(x)\epsilon^{abc} \ . \tag{1.1}$$

One then computes the correlation function

$$\pi(p^2)=i\int d^4x\, e^{ip\cdot x}\langle 0\,|\,T(\eta(x)\overline{\eta}(0))\,|\,0\rangle \tag{1.2}$$

which satisfies a dispersion relation of the form

$$\pi(p^2)=\frac{1}{\pi}\int \frac{\mathrm{Im}\pi(p'^2)}{p'^2-p^2-i\epsilon}dp'^2 \ . \tag{1.3}$$

For large p^2, that is in the limit $x\rightarrow 0$, the product $\eta(x)\overline{\eta}(0)$ can be computed in terms of the quark and gluon degrees of freedom via the operator-product expansion (OPE). This in turn leads to an expansion of $\pi(p^2)$ in terms of the various vacuum correlation functions, such as the chiral-symmetry-breaking parameter $\langle \overline{q}q \rangle$. On the other hand, using a dispersion relation, the correlation $\pi(p^2)$ can be computed as an integral over the absorptive part, that is to say, in terms of the nucleon and excited states which have the same quantum number as the nucleon, apart from parity. By matching the Borel transforms of these two calculations, one in terms of the operator-product expansion and the other in terms of the physical intermediate states, over a range of values of the Borel mass parameter in the region of the nucleon mass, one is able to deduce self-consistently the proton mass and the coupling strength of the current η to the one-proton

$$\langle 0 | T(\eta(x)\bar{\eta}(0)) | 0 \rangle = f(x^2)\hat{x} + g(x^2)1$$

$$\pi(p^2) = F(p^2)\hat{p} + G(p^2)1$$

From f(x) by FT get f(p). f(p) has a OPE valid for large p . F(p) has an expansion in term of physical intermediate terms . Borel transform f(p) and F(p) and match the two to obtain QCD sum rule. We see some details a bit later. Note also F(p) and G(p) independent functions , F(p) is chiral odd while G(p) chiral even. So states of opposite parity add in F(p) while in G(p) , they contribute with opposite signs.

We shall follow closely the notations of Ioffe [14]. We consider the nucleon correlator in an external field

$$\Pi(p, A_\mu) = i \int d^4x \; e^{ip.x} \langle 0 | T\{\eta(x), \bar{\eta}(0)\} | 0 \rangle \Big|_{A_\mu} \qquad (26)$$

where $\eta(x)$ is the nucleon current

$$\eta(x) = \epsilon^{abc} u^a(x) \; C\gamma_\mu u^b(x) \gamma^\mu \gamma_5 d^c(x) \qquad (27)$$

with proton quantum numbers; u^a, d^b are quark fields and a, b, c are color indices. A_μ refers to constant external field. To compute the matrix element of the current j^5_μ between a proton state $\langle p | j^5_\mu | p \rangle$ one adds a term

$$\Delta\mathcal{L} = j^5_\mu A^\mu \qquad (28)$$

•The various matrix elements like vector , axial vector etc . By studying the propagation is an appropriate external field

•

$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d \qquad \text{isovector} \quad (29)$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d \qquad \text{isoscalar} \quad (30)$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s \qquad \text{octet} \quad (31)$$
$$j^5_\mu = \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d + \bar{s}\gamma_\mu\gamma_5 s \qquad \text{SU(3) singlet} \quad (32)$$

$$\langle p,s|\,\frac{4}{9}\bar{u}\gamma_\mu\gamma_5 u+\frac{1}{9}\bar{d}\gamma_\mu\gamma_5 d+\frac{1}{9}\bar{s}\gamma_\mu\gamma_5 s\,|p,s\rangle=-s_\mu G_{Bj}$$

$$\begin{aligned} G_{Bj} \;=\; & \frac{1}{6}(G_U-G_D)+\frac{1}{3}(G_U+G_D) \\ & -\frac{1}{18}(G_U+G_D-2G_S) \end{aligned}$$

$$G_U-G_D=1.267$$

$$G_U+G_D-2G_S=0.585$$

Ellis and Jaffe

$$G_{\mathrm{Bj}}=\frac{1}{6}G_A+\frac{5}{18}G_8$$

- Internal inconsistency of Ellis Jaffe

[5] D.J.Gross, S.B.Treiman and F.Wilczek Phys.Rev. D **19**

We briefly recall the argument of ref [5]. If we ignore the anomaly then we have, for the divergence of isoscalar current

$$\partial^{\mu}\left[\bar{u}\gamma_{\mu}\gamma_{5}u+\bar{d}\gamma_{\mu}\gamma_{5}d\right] = i(m_u+m_d)\left[\bar{u}\gamma_5 u+\bar{d}\gamma_5 d\right] + i(m_u-m_d)\left[\bar{u}\gamma_5 u-\bar{d}\gamma_5 d\right] \quad (7)$$

$$\langle N|\partial^{\mu}(\bar{u}\gamma_{\mu}\gamma_{5}u+\bar{d}\gamma_{\mu}\gamma_{5}d)|N\rangle_{I=1} = \frac{m_u-m_d}{m_u+m_d}\langle N|\partial^{\mu}(\bar{u}\gamma_{\mu}\gamma_{5}u-\bar{d}\gamma_{\mu}\gamma_{5}d)|N\rangle_{I=1}$$

$$\langle N|\mathcal{O}|N\rangle_{I=1} = \langle p|\mathcal{O}|p\rangle - \langle n|\mathcal{O}|n\rangle.$$

This conclusion is avoided by noting that one has ignored the anomaly. In Eqn.(7) one should write

$$\begin{aligned}\partial^\mu\left[\bar u\gamma_\mu\gamma_5 u+\bar d\gamma_\mu\gamma_5 d\right] &= i(m_u+m_d)\left[\bar u\gamma_5 u+\bar d\gamma_5 d\right]\\ &+ i(m_u-m_d)\left[\bar u\gamma_5 u-\bar d\gamma_5 d\right]\\ &+ 2\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde G^a_{\mu\nu} \qquad (10)\end{aligned}$$

where $\tilde G^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}$. Using a Sutherland type argument it is derived in ref [5]

$$i\langle 0|(m_u\bar u\gamma_5 u+m_d\bar d\gamma_5 d)|\pi^0\rangle+\langle 0|\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde G^a_{\mu\nu}|\pi^0\rangle=0$$

where, $q=s,\ c,\dots$ etc. Using again a PCAC argument they[5] obtain

$$\begin{aligned}2i\langle 0|m_u\ \bar u\gamma_5 u+m_d\ \bar d\gamma_5 d|\pi^0\rangle &= \frac{m_u-m_d}{m_u+m_d}\ F_\pi\ m_\pi^2\sqrt{2}\\ &= -2\langle 0|\frac{g^2}{16\pi^2}G^a_{\mu\nu}\tilde G^a_{\mu\nu}|\pi^0\rangle\end{aligned}$$

$$\langle 0|\ \frac{3\alpha_s}{4\pi}\ G^a_{\mu\nu}\tilde G^a_{\mu\nu}|\eta\rangle=\sqrt{\frac{3}{2}}F_\pi m_\eta^2$$

In the presence of an external field vacuum is not Lorentz invariant. New vacuum matrix elements must be added

$$\langle 0|\bar{q}\gamma_\mu\gamma_5 q|0\rangle|_{A_\mu} \ = \ FA_\mu$$

$$\Pi^{I=0}_{\mu\nu} \ = \ \frac{i}{2}\int d^4x e^{iq.x}$$
$$\langle 0|T(\bar{u}(x)\gamma_\mu\gamma_5 u(x) + \bar{d}(x)\gamma_\mu\gamma_5 d(x),$$
$$\bar{u}(0)\gamma_\nu\gamma_5 u(0) + \bar{d}(0)\gamma_\nu\gamma_5 d(0))|0\rangle$$

with

$$F(I=0) = -\Pi^{I=0}_1(q^2=0)$$

We are in interested in the value at q=0. This can be valuated either directly using a sum involving spin one axial vector mesons or alternately one can take the divergence . First singlet current is not conserved because of the anomaly and

secondly sum rule now involves pseudoscalar states . Both methods give consistent results and we can successfully account for the experimental value by bjorken sum rule.

$$G = G_U + G_D = 0.22$$

$$\begin{aligned} G_{Bj} &= \frac{1}{6}(G_U - G_D) + \frac{1}{3}(G_U + G_D) \\ &\quad -\frac{1}{18}(G_U + G_D - 2G_S) \end{aligned}$$

$$G_U - G_D = 1.267$$

$$G_U + G_D - 2G_S = 0.585$$

$$G_U + G_D = 0.22$$

$$G_{Bj}(\mu^2 = 1\ GeV^2) = 0.32$$

to be compared with experimental value [3]

$$G_{Bj}(\mu^2 = 5\ GeV^2) \approx 0.28,$$

[3] D. Adams et.al. Phys. Rev. D **56** 5330 (1997)

Following Ioffe [12-15], we introduce the correlator of axial vector currents

$$\Pi_{\mu\nu}^{ab}(q) = \mathrm{i}\int d^4 x e^{iqx} \langle 0| T \left\{ J_{\mu 5}^{a}(x), J_{\nu 5}^{b}(0) \right\} |0\rangle ; \quad (a,\mathrm{b}=8,0). \qquad \ldots..(17)$$

The general form of the polarization tensor $\Pi_{\mu\nu}^{ab}(q)$ is

$$\Pi_{\mu\nu}^{ab}(q) = -\mathrm{P}_{L}^{ab}(q^2)\,\mathrm{g}_{\mu\nu} + \mathrm{P}_{T}^{ab}(q^2)\,(-\mathrm{q}^2\,\mathrm{g}_{\mu\nu} + \mathrm{q}_{\mu}\mathrm{q}_{\nu}) \qquad \ldots.(18)$$

The functions $\mathrm{P}_{L}^{ab}(q^2)$ and $\mathrm{P}_{T}^{ab}(q^2)$ are free from kinematic singularities. On forming the divergence with the momentum, we get

$$\mathrm{q}^{\mu}\,\Pi_{\mu\nu}^{ab}(q)\,\mathrm{q}^{\nu} = -\mathrm{P}_{L}^{ab}(q^2)\,\mathrm{q}^2 \qquad \ldots.(19)$$

On the other hand from Eq (17) we have the Ward identity [14]

$$q^{\mu}\ \Pi^{00}_{\mu\nu}(q)\ q^{\nu} = i36 \int d^4x e^{iqx} \langle 0 | T\{Q_5(x), Q_5(0)\} | 0 \rangle + i6 \int d^4x e^{iqx} \langle 0 | T\{Q_5(x), D(0)\} | 0 \rangle$$

$$+ i6 \int d^4x e^{iqx} \langle 0 | T\{D(x), Q_5(0)\} | 0 \rangle$$

$$+ i \int d^4x e^{iqx} \langle 0 | T\{D(x), D(0)\} | 0 \rangle + 4\sum_{i=u,d,s} m_i \langle 0 | \bar{q}_i q_i | 0 \rangle \qquad .....(20)$$

In Eq.(20) we have introduced the notation

$$Q_5(x) = (\alpha_s/8\pi)\ G^a_{\mu\nu}(x)\tilde{G}^{a\mu\nu}(x) \qquad ....(21)$$

$$D(x) = 2i \sum_{i=u,d,s} m_i \bar{q}_i(x)\gamma_5 q_i(x) \qquad ...(22)$$

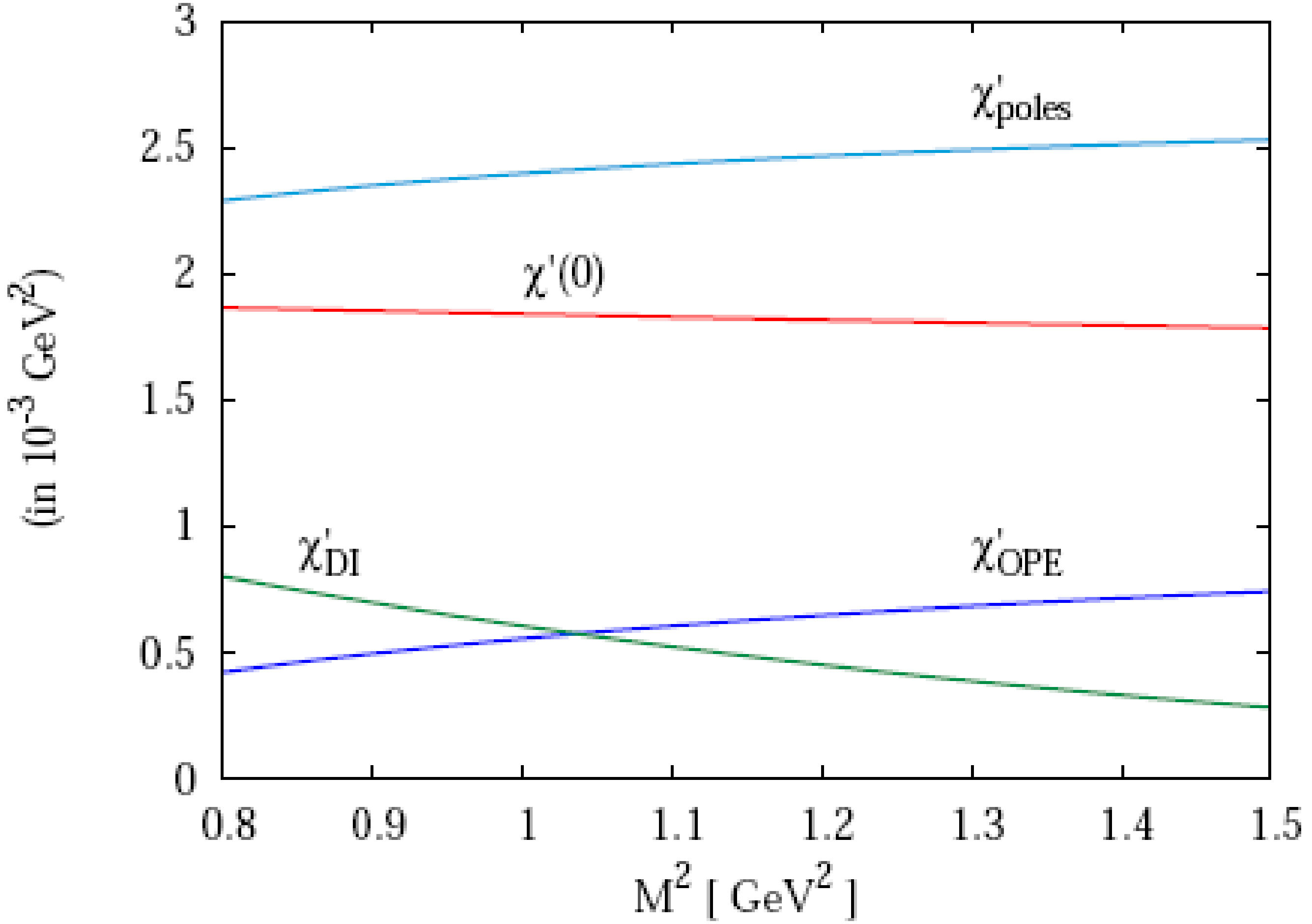
$$\chi(q^2) = \mathrm{i}\left(\frac{\alpha_s}{4\pi}\right)^2 \int d^4x e^{iqx} \langle 0|T\left\{G\tilde{G}(x), G\tilde{G}(0)\right\}|0\rangle$$

$$\left\langle 0\right| J^a_{\mu 5} \left|P(p)\right\rangle = if^a_P p_\mu ,$$

$$\begin{pmatrix} f^8_\eta & f^0_\eta \\ f^8_{\eta'} & f^0_{\eta'} \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_8 & -f_0 \sin\theta_0 \\ f_8 \sin\theta_8 & f_0 \cos\theta_0 \end{pmatrix}$$

$$\begin{aligned} \langle 0|\frac{3\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta\rangle &= \sqrt{\frac{3}{2}}\, m^2_\eta \left(f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0\right) \\ \langle 0|\frac{3\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}|\eta'\rangle &= \sqrt{\frac{3}{2}}\, m^2_{\eta'} \left(f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0\right) . \end{aligned}$$

$$
\begin{aligned}
\chi'(0) \;=\;& \frac{f_\pi^2}{8}\left(\frac{m_d - m_u}{m_d + m_u}\right)^2 \left(1 + \frac{m_\pi^2}{M^2}\right) e^{\frac{-m_\pi^2}{M^2}} + \frac{1}{24}\left(f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0\right)^2 \left(1 + \frac{m_\eta^2}{M^2}\right) e^{\frac{-m_\eta^2}{M^2}} \\
& + \frac{1}{24}\left(f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0\right)^2 \left(1 + \frac{m_{\eta'}^2}{M^2}\right) e^{\frac{-m_{\eta'}^2}{M^2}} \\
& - \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{1}{\pi^2} M^2 E_0(W^2/M^2) \left[1 + \frac{\alpha_s}{\pi}\frac{74}{4} + \frac{\alpha_s}{\pi}\frac{9}{2}\left(\gamma - \ln\frac{M^2}{\mu^2}\right)\right] \\
& - 16 \left(\frac{\alpha_s}{4\pi}\right)^3 \frac{1}{M^2} \sum_{i=u,d,s} m_i \langle \bar{q}_i q_i \rangle - \frac{9}{64}\frac{1}{M^2}\left(\frac{\alpha_s}{\pi}\right)^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \\
& + \frac{1}{16}\frac{1}{M^4}\frac{\alpha_s}{\pi}\left\langle g_s \frac{\alpha_s}{\pi} G^3 \right\rangle - \frac{5}{128}\frac{\pi^2}{M^6}\frac{\alpha_s}{\pi}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle^2 . \qquad (20)
\end{aligned}
$$


$\chi'_{\rm poles}$
$\chi'(0)$
$\chi'_{\rm DI}$
$\chi'_{\rm OPE}$
(in 10^{-3} GeV^2)
M^2 [GeV^2]
0
0.5
1
1.5
2
2.5
3
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5

$$
\begin{aligned}
\frac{1}{12} f_{\eta_\chi}^2 \left(1+\frac{m_\chi^2}{M^2}\right) e^{\frac{-m_\chi^2}{M^2}} \quad \approx \quad & \frac{1}{24} f_\pi^2 \left(1+\frac{m_\pi^2}{M^2}\right) e^{\frac{-m_\pi^2}{M^2}} \\
+ \quad & \frac{1}{24}\left(f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0\right)^2 \left(1+\frac{m_\eta^2}{M^2}\right) e^{\frac{-m_\eta^2}{M^2}} \\
+ \quad & \frac{1}{24}\left(f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0\right)^2 \left(1+\frac{m_{\eta'}^2}{M^2}\right) e^{\frac{-m_{\eta'}^2}{M^2}} .
\end{aligned}
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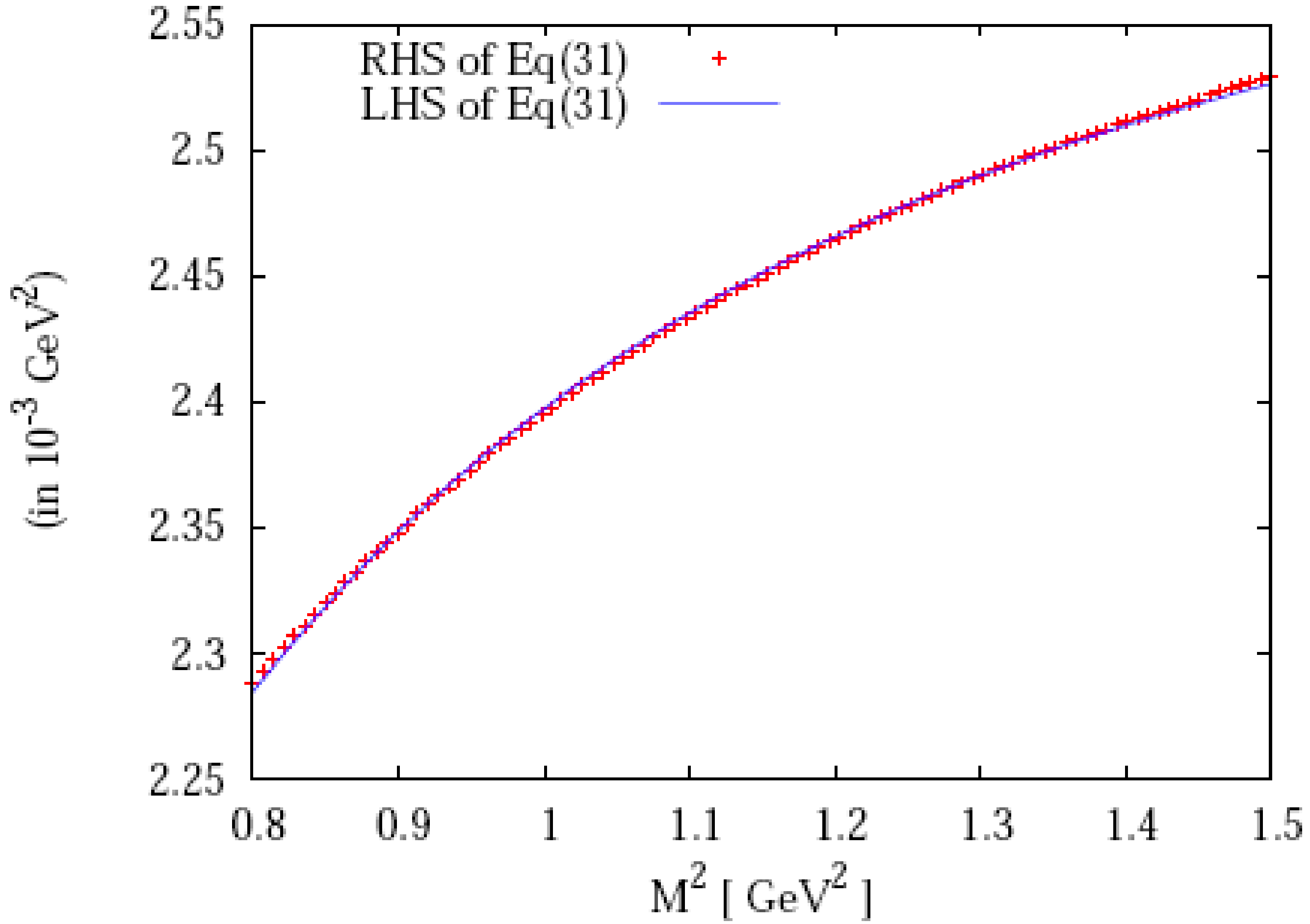

Fig. 2: Estimate of η' mass and coupling in the chiral limit, see Eq.(31). The continuous curve corresponds to $m_\chi = 723$ MeV.

For $\chi'(0)$ we obtained a value $\approx 1.82\times10^{-3}$ GeV^2.

This is consistent with

1.9×10^{-3} GeV^2 obtained in Ref.[18] using only the axial vector current sum rules.

Further, Leutwyler [19] in chiral perturbation theory found that

$$\chi'(0) = \frac{F^2}{4} m_{red}^2 \left[\frac{1}{m_u^2} + \frac{1}{m_d^2} + \frac{1}{m_s^2}\right] + \frac{1}{2} H_0 \qquad \ldots(13)$$

where

$$(m_{red})^{-1} = m_u^{-1} + m_d^{-1} + m_s^{-1}, \qquad \ldots.(14)$$

H_0 is a parameter in the effective chiral Lagrangian that describes low energy QCD and is expected to be small. Leutwyler estimates the first term in Eq.(13) which depends only on quark mass ratios and not on their absolute values to be 2.2×10^{-3} GeV^2, which is consistent with the determination in Ref.[13,14,17,18].

In Ref.[17], we had estimated the mass of η′ in the chiral limit to be

$m_{\eta'}(m_q = 0) = 723$ MeV, …(15a)

$F_0(m_q = 0) = 178$ MeV. …(15b)

Returning to Witten's formula Eq.(7), let us note that $\chi(0)\,|_{GD}$ has been determined in the lattice to be [20]:

$\chi(0)\,|_{GD} = (191\pm5)^4$ MeV^4. … (16a)

On the other hand , we can determine $\chi(0)\,|_{GD}$ using Eq.(7) and Eqs. (15a) and (15b) above, which gives

$\chi(0)\,|_{GD} = (193 \text{ MeV})^4$ …(16b)

in excellent agreement with the lattice value Eq. (16a)

It was shown by Witten [3] using $1/N_c$ expansion, where N_c is the number of colors, that

$$m_\chi^2 = 12\,\chi(0)\,|_{GD} \,/\, F^2 \quad ..$$

In continuation of these J.P.Singh and I have determining the couplings of eta and etaprime to octet and singlet currents in a recent work. A number of functions, like the longitudinal current correlations , topological susceptibility ,pseudoscalar density correlations have been studied.

$$\Pi_{\mu\nu}^{ab}(q) = \mathrm{i}\int d^4x e^{iqx} \langle 0 | T \left\{ J_{\mu 5}^{a}(x), J_{\nu 5}^{b}(0) \right\} | 0 \rangle ; \quad (a,b=8,0).$$

The general form of the polarization tensor $\Pi_{\mu\nu}^{ab}(q)$ is

$$\Pi_{\mu\nu}^{ab}(q) = -\mathrm{P}_{L}^{ab}(q^2)\,\mathrm{g}_{\mu\nu} + \mathrm{P}_{T}^{ab}(q^2)(-\mathrm{q}^2\mathrm{g}_{\mu\nu} + \mathrm{q}_\mu \mathrm{q}_\nu)$$

$$\chi(q^2) = \mathrm{i}\left(\frac{\alpha_s}{4\pi}\right)^2 \int d^4x e^{iqx} \langle 0 | T \left\{ G\tilde{G}(x), G\tilde{G}(0) \right\} | 0 \rangle$$

$$\mathrm{S}(\mathrm{q}^2) = \mathrm{i}\int d^4x e^{iqx} \langle 0 | \mathrm{T}\left\{ m_s \bar{s}(x)\gamma_5 s(x), m_s \bar{s}(0)\gamma_5 s(0) \right\} | 0 \rangle$$

$$P_L^{88}(q^2),\ \mathrm{S}(\mathrm{q}^2),\ P_L^{08}(q^2),\ P_L^{00}(q^2) - 12\frac{\chi(q^2)}{q^2},\ \frac{\chi'(q^2)}{q^2},\ P_L^{00}(q^2) \text{ and } \frac{\chi(q^2)}{q^2}$$

Comparison of our results on couplings and mixing angles with those obtained by other authors.

Ref.	Specification	f_8 (MeV)	f_0 (MeV)	θ_8 (Degree)	θ_0 (Degree)
This work	Sum rules , using 4.5 4.11 cf. also Table I	176.2	154.6	−21.6	−6.3
[26]	f_8 from [8] f_8 from [7] best fit phen.	1.28 f_π=167.3 1.34 f_π=175.5 (1.51 ± 0.05) f_π $=197.4 \pm 6.5$	154.23 ± 5.2 156.84 ± 5.2 168.60 ± 5.2	$-(22.2 \pm 1.8)$ $-(22.9 \pm 1.8)$ $-(23.8 \pm 1.4)$	$-(8.7 \pm 2.1)$ $-(6.9 \pm 2.0)$ $-(2.4 \pm 1.9)$
[27, 28]	Theory Phen.	155.53 ± 7.8 164.68 ± 7.8	143.77 ± 5.2 152.92 ± 5.2	$-(19.4 \pm 1.4)$ $-(21.2 \pm 1.4)$	$-(6.8 \pm 1.4)$ $-(9.2 \pm 1.4)$
[8]	ChPT	167.30	143.77	−20.5	−4.0
[29]	ChPT	172.53	164.05	−20.0	-1.0 ± 1.5
[16]	Sum rules	188.21	176.45	−8.4	−13.8
[15]	Sum rules		178 ± 17	$-(17.0 \pm 5.0)$	
[10]	Extended current algebra	148.0	150.7	−20.1	−12.3

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$$\langle 0 | \eta | P_j \rangle = \lambda_j^+ v(m_j^+) ,$$

and

$$\langle 0 | \eta | S_j \rangle = \lambda_j^- \gamma_5 v(m_j^-) ,$$

$$\frac{M^6}{L^{4/9}} + \frac{bM^2}{4L^{4/9}} + \tfrac{4}{3} a^2 L^{4/9} - \frac{a^2 m_0^2}{3M^2} = 2(2\pi)^4 \left[\sum_j (\lambda_j^+)^2 e^{-m_j^{+2}/M^2} + \sum_j (\lambda_j^-)^2 e^{-m_j^{-2}/M^2} \right] .$$

The structure at 1,

$$2aM^4 - \frac{ab}{9} + \frac{8\times 17}{81} \frac{\alpha_s}{\pi} \frac{a^3}{M^2} = 2(2\pi)^4 \sum_j [m_j^+ (\lambda_j^+)^2 e^{-(m_j^+)^2/M^2} - m_j^- (\lambda_j^-)^2 e^{-m_j^{-2}/M^2}] .$$

A forteriori, the OPE on

the left-hand side becomes more and more accurate for larger values of M^2, therefore the behavior of the sum over the physical states on the right-hand side is more exactly described for large M^2 by the leading term on the left-hand side. Hence the only way by which the two sum rules can be consistent is for the excited state contributions in the right-hand side of the second sum rule to cancel asymptotically. In particular it strongly suggests that $| \lambda_j^{+2} - \lambda_j^{-2} | \to 0$ and $| m_j^{+2} - m_j^{-2} | \to 0$ for large j.

Then it is reasonable to expect

the excited states, chiral symmetry is realized in the Wigner-Weyl mode,

$$m_j^+ \approx m_j^- , \quad \lambda_j^+ \approx \lambda_j^- ,$$

- Review for R.L.Jaffe et.al hep-ph/0602010 , Barger and Cline , Glozman

- Assorted Topics in QCD Sum Rules and Nucleon Properties

J Pasupathy

January 17, 2009

Some of the topics presented in the talk are discussed in detail in the following. There have crises about proton properties, spin, electromagnetic mass difference with neutron strangeness content. These have disappeared over the years.

• L. Ioffe, Nucl. Phys. B188, 317 (1981); B191, 591(E) (1981); V.Belyaev andB. L. Ioffe, Sov. Phys. JETP56, 493 (1982); B. L. Ioffe and A. V. Smilga, Nucl. Phys, B232, 109 (1984); B. L. Ioffe, hep-ph/9511401, hep-ph/9804328; hep-ph/9901223.

• Baryon magnetic moments, Phys. Rev. D33 p.1961, (1986); D36 p.1553 (1987).

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- Bjorken Sum Rule Phys. Rev. D32 p.1786 (1985); D33 p.2597 (1989); hep-th/0312304, Int. J. Mod. Phys. A21 (2006) 5099-5118.

  Neutron-Proton Mass Difference, Nolan-Schiffer Anomaly, Phys. Rev D51 p. 3688 (1995).

- Derivative of Topological Susceptibility $\chi'(0)$ and η' Mass in the Chiral Limit, Phys. Lett. B634 p.508 (2006).

- Axial and Pseudoscalar Correlators and their Coupling to η and η , J. P. Singh, J. P., to be published.

- Parity Doubling and Wigner-Weyl Mode of Chiral Symmetry Realisation, Phys. Rev. D32 p.1786 (1985).

- Anomalies, Symmeries and Strangeness content of the proton, Pramana 61, 943 (2003).

- One can use QCD sum rules also to learn properties of excited states as follows: one must fix the ground state mass and coupling from experiments as well as the mass of the excited states. Then determine the coupling of the excited states accurately from sum rules. In the case of charmonium, nclusion of higher order terms in OPE improves agreement with experiments.
- My apologies for the poor quality of the slides, which are obtained from the listed above papers using cut-and-paste.