# Duality violations in hadronic tau decays and $\alpha_s$

(How well do we know the strong coupling constant?)

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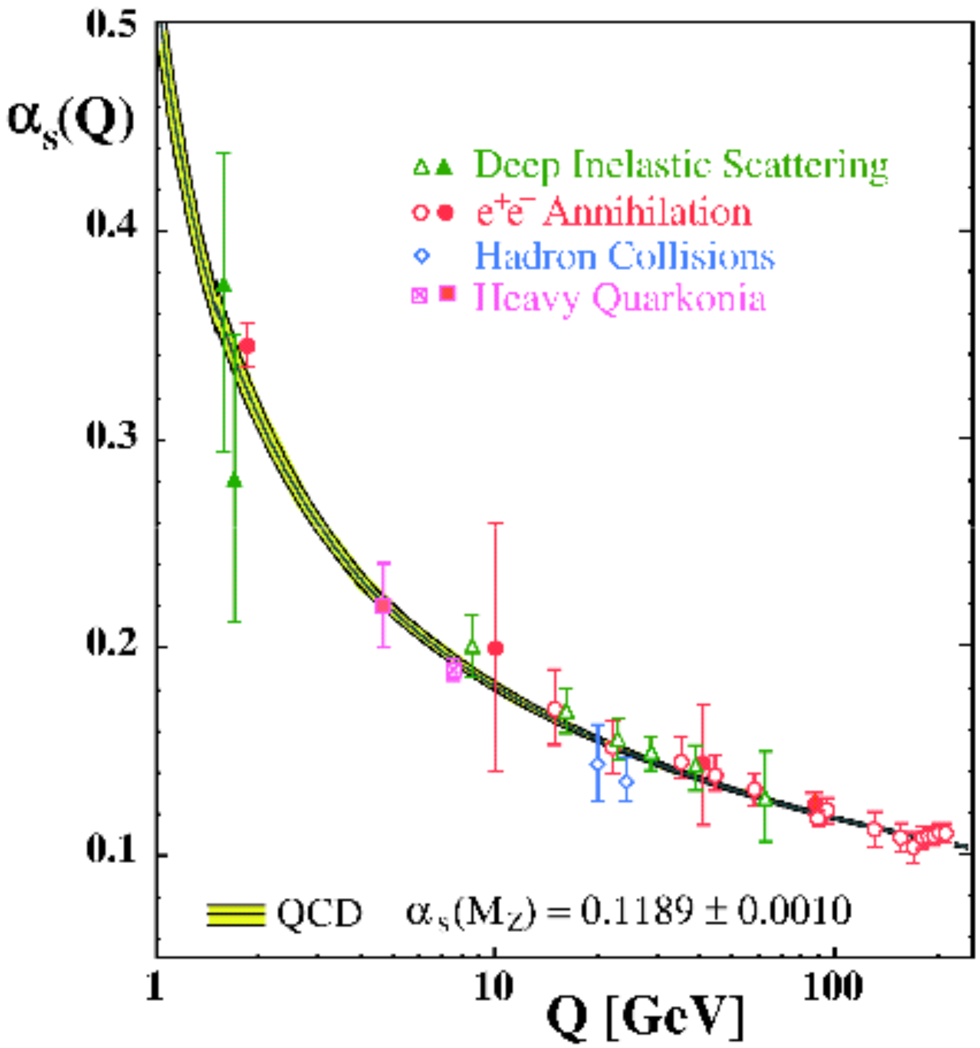
# Outline

- Current status of determination of $\alpha_s$
- Non-perturbative effects in $\tau$ decays and duality violations: quarks versus hadrons
- An *ansatz* for duality violations
- Results
- Outlook




(from Bethke, 2006)

- $Q$ = 1.78 GeV for $\tau$ decay
- curve: most precise perturbative calculation through $\alpha_s(Q=M_Z)$
- confirms QCD prediction
- misses $\tau$ point by a little?

# Value(s) of $\alpha_s$

world average: $\alpha_s(M_Z) = 0.1189 \pm 0.0010$ (Bethke, 2006)

$\tau$ decay: $\alpha_s(m_\tau) = 0.345 \pm 0.004_{exp} \pm 0.009_{th}$ (ALEPH, 2006)
$\rightarrow$ $\alpha_s(M_Z) = 0.1215 \pm 0.0012$

lattice: $\alpha_s(7.5\ \text{GeV}) = 0.2082 \pm 0.0040$ (Mason et al., 2005)
(bottonium) $\rightarrow$ $\alpha_s(M_Z) = 0.1170 \pm 0.0012$

These values are 2.6 standard deviations apart.

world average without lattice: $\alpha_s(M_Z) = 0.1200 \pm 0.0014$ (Bethke, 2006)
$\rightarrow$ lattice is 1.6 $\sigma$ lower
world average without $\tau$ decay: $\alpha_s(M_Z) = 0.1176 \pm 0.0018$
$\rightarrow$ $\tau$ decay is 1.8 $\sigma$ higher

$m_\tau = 1.776$ GeV << 7.5 GeV $\rightarrow$ non-perturbative effects more likely?

## Recent updates on the value of $\alpha_s$

τ decay: $\alpha_s(m_\tau) = 0.344 \pm 0.005_{exp} \pm 0.007_{th}$ (Davier et al., 2008)

$\rightarrow \alpha_s(M_Z) = 0.1212 \pm 0.0011$

lattice: $\alpha_s(7.5\ \text{GeV}) = 0.2120 \pm 0.0028$ (Davies et al., 2008,

(bottonium) $\rightarrow \alpha_s(M_Z) = 0.1183 \pm 0.0008$ see also Maltman et al., 2008)

These values are 2.1 standard deviations apart.

New analyses of tau decays:

$\alpha_s(m_\tau) = 0.332 \pm 0.005_{exp} \pm 0.015_{th}$ (Baikov et al., 2008)

(order $\alpha_s^4$)

$\alpha_s(m_\tau) = 0.316 \pm 0.003_{exp} \pm 0.005_{th}$ (Beneke & Jamin, 2008)

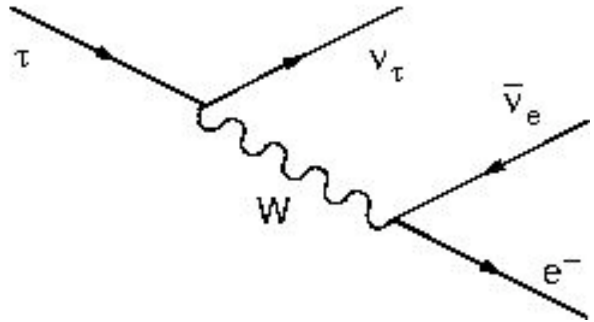
(FOPT plus Borel sum)

$\alpha_s(m_\tau) = 0.321 \pm 0.005_{exp} \pm 0.012_{th}$ (Maltman & Yavin, 2008)

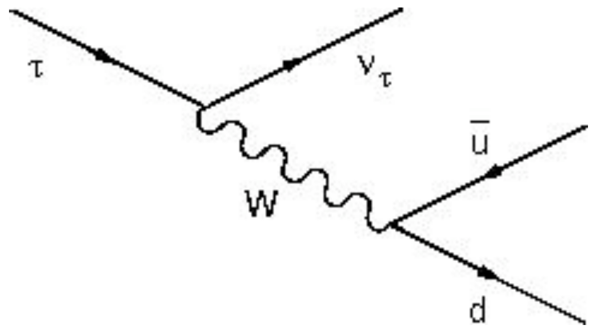
(higher order in OPE, different weights)

# $\tau$ decays

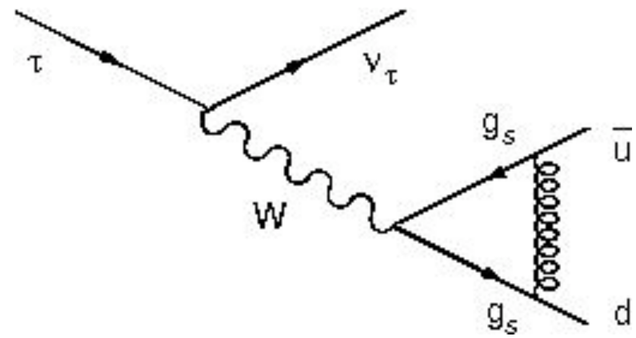
leptonic:




hadronic:




+

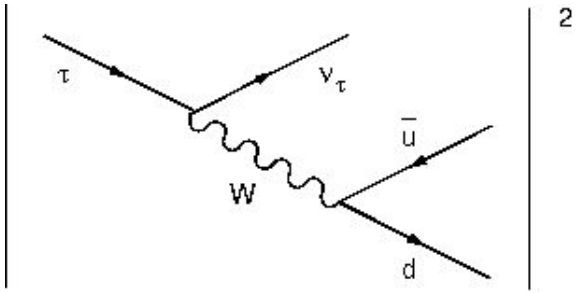


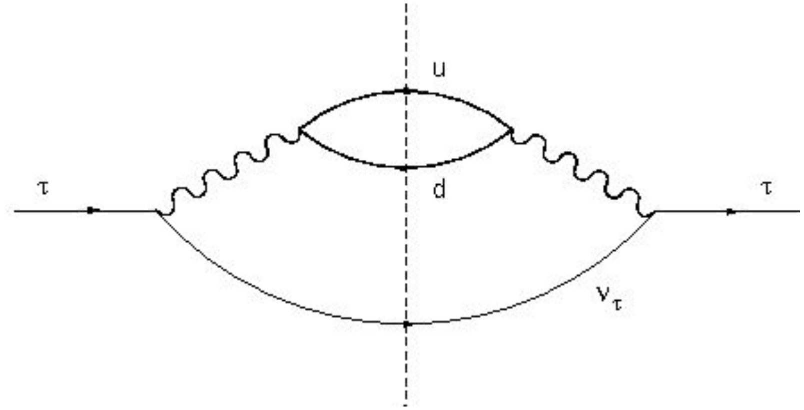

$$R_\tau = \frac{\Gamma(\tau \to \nu_\tau \text{ hadrons})}{\Gamma(\tau \to \nu_\tau e \overline{\nu}_e)} = 3S_{EW}\left[1 + \frac{\alpha_s}{\pi} + 5.2\left(\frac{\alpha_s}{\pi}\right)^2 + \dots\right]$$

- use this to determine $\alpha_s(m_\tau)$ ($p_W \sim m_\tau$)
- see $\tau \to \nu_\tau \rho^* \to \nu_\tau$pions, not $\tau \to \nu_\tau$jets
  $\rho^* = \rho(770), \rho(1450), \rho(1700)$ (and others)

$\to$ relation with perturbative regime?

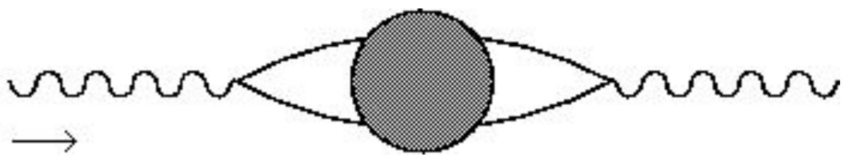
# $\Gamma(\tau \rightarrow \nu_\tau \text{ hadrons})$

$\propto$ 


$\propto$ Im 


(optical theorem)

$$R_\tau = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi(s)$$

$\Pi(s)$ = 


$s = (p_W)^2$

$0 \leq s \leq m_\tau^2$ depending on how much momentum $\nu_\tau$ carries away

Determine $\alpha_s$ by equating

$$\begin{aligned} R_\tau &= 3S_{EW}\left[1+\frac{\alpha_s}{\pi}+5.2\left(\frac{\alpha_s}{\pi}\right)^2+\ldots\right] \\ &= 12\pi S_{EW}\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi(s) \end{aligned}$$

with $\mathrm{Im}\,\Pi(s)$ from experiment.

But: worry about the … : usually done through the OPE:

$$\Pi(s)\approx\Pi_{OPE}(s)=-\frac{1}{4\pi^2}\log\left(\frac{-s}{\mu^2}\right)\left(1+\frac{\alpha_s(\mu^2)}{\pi}+\ldots\right)+\frac{c_2}{s}+\frac{c_4}{s^2}+\ldots$$

$c_2$ : proportional to quark mass squared -- very small

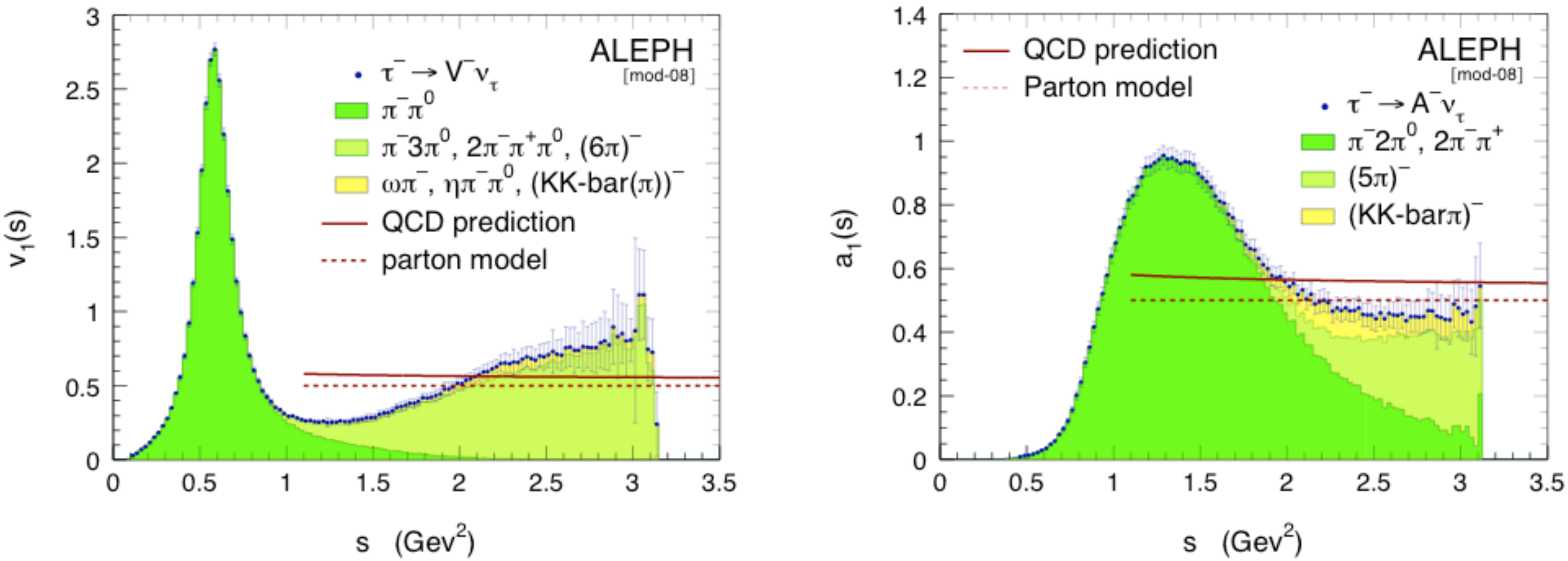
$c_4$ : quark mass times chiral condensate (use PCAC) plus gluon condensate, etc.

ALEPH 2008:

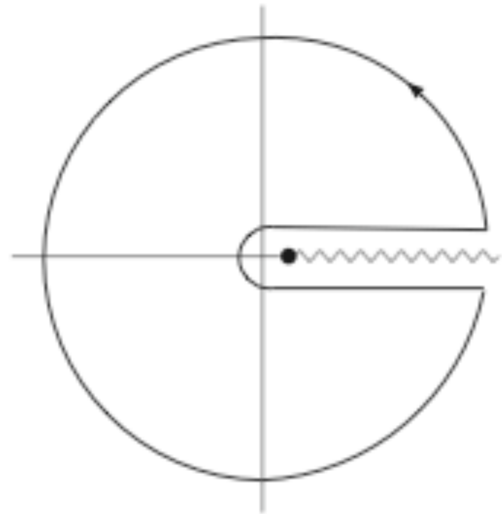
Gluon condensate from V: $(\alpha_s/\pi)\langle GG\rangle$ = -0.008(4) GeV$^2$

from A: = -0.022(4) GeV$^2$

(2.5 standard deviations apart)



# Relating $\alpha_s$ to $\tau$ decay data -- duality violations:



complex $s$ plane
circle has radius $m_\tau^2$

- positive real axis: spectral data (Im $\Pi(s)$)
- perturbation theory + OPE valid for large $|s|$ anywhere but there!
- standard analysis: $\Pi(s)$ analytic everywhere except for real $s > 0$

(Braaten, Narison and Pich, 1992)

$\rightarrow$ Cauchy:

$$\int_0^{m_\tau^2} ds\, P(s)\, \frac{1}{\pi}\, \mathrm{Im}\, \Pi(s) = -\frac{1}{2\pi i} \oint_{|s|=m_\tau^2} ds\, P(s)\, \Pi(s)$$

Take $$P_{k\ell}(s) = \left(1 - \frac{s}{m_\tau^2}\right)^{2+k} \left(1 + 2\frac{s}{m_\tau^2}\right) \left(\frac{s}{m_\tau^2}\right)^{\ell}$$

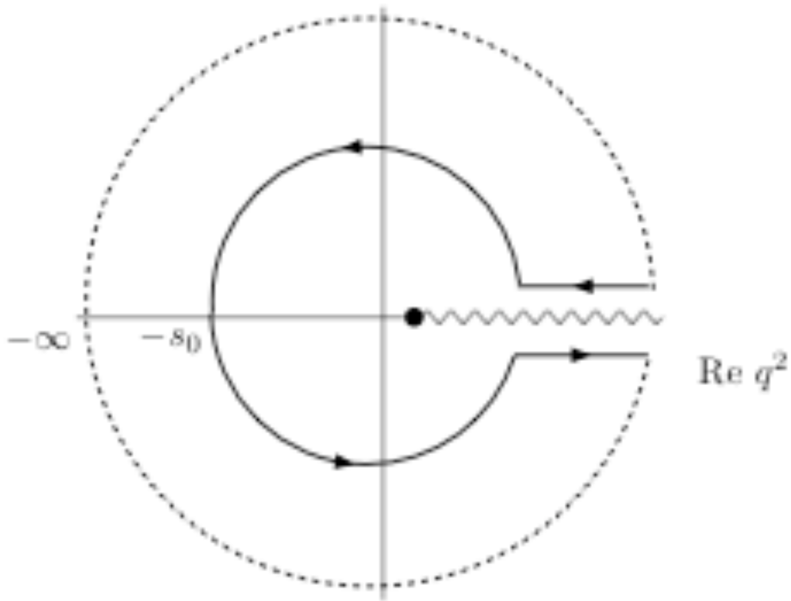
and approximate (Le Diberder and Pich, 1992)

$$\Pi(s) \approx \Pi_{OPE}(s) = -\frac{1}{4\pi^2} \log\left(\frac{-s}{\mu^2}\right) \left(1 + \frac{\alpha_s(\mu^2)}{\pi} + \dots\right) + \frac{c_2}{s} + \frac{c_4}{s^2} + \dots$$

then $R_{kl}$ =

$$\int_0^{m_\tau^2} ds\, P_{k\ell}(s) \frac{1}{\pi} \mathrm{Im}\,\Pi(s) = -\frac{1}{2\pi i} \oint_{|s|=m_\tau^2} ds\, P_{k\ell}(s)\, \Pi_{OPE}(s) + \mathcal{D}_{k\ell}(m_\tau^2)$$

- $P_{kl}(s)$ suppresses region near $s = m_\tau^2$ → ignore “duality violations”
- fit $\alpha_s$ , $c_4$ , $c_6$ , $c_8$ using this equation for *kl* = 00, 10, 11, 12, 13, ignoring duality violations ($c_2$ is available from theory)
- duality violations $\mathcal{D}_{k\ell}(m_\tau^2)$ probably exponentially suppressed, but what is their quantitative effect on the analysis?




Assume:

- OPE asymptotic;
  applies except on
  Minkowski axis

- $\Delta(q^2)$ decays exponentially
  for $|q^2| \to \infty$

Then

$$\mathcal{D}^P(s_0) = -\frac{1}{2\pi i}\oint_{|q^2|=s_0} dq^2\, P(q^2)\Delta(q^2) = -\int_{s_0}^{\infty} ds\, P(s)\frac{1}{\pi}\,\mathrm{Im}\,\Delta(s)$$

$$\Pi(q^2) = \Pi_{OPE}(q^2) + \Delta(q^2)$$

Need *ansatz* for the non-OPE part of the spectral function for $s > s_0$ .

*Ansatz*: $$\frac{1}{\pi}\,\mathrm{Im}\,\Delta(s) = \kappa\, e^{-\gamma s}\sin(\alpha+\beta s)$$

Oscillatory: duality violations due to resonances

Exponential decay: finite width $\Rightarrow$ $\gamma \propto 1/N_c$ , small

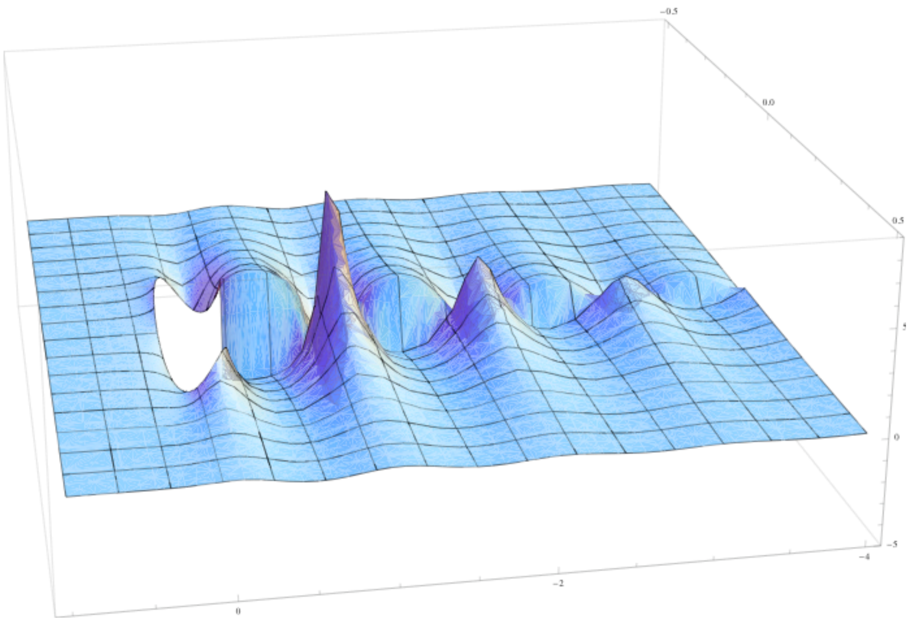
Argument of sine linear in $s$ : Regge-like (daughter) trajectories

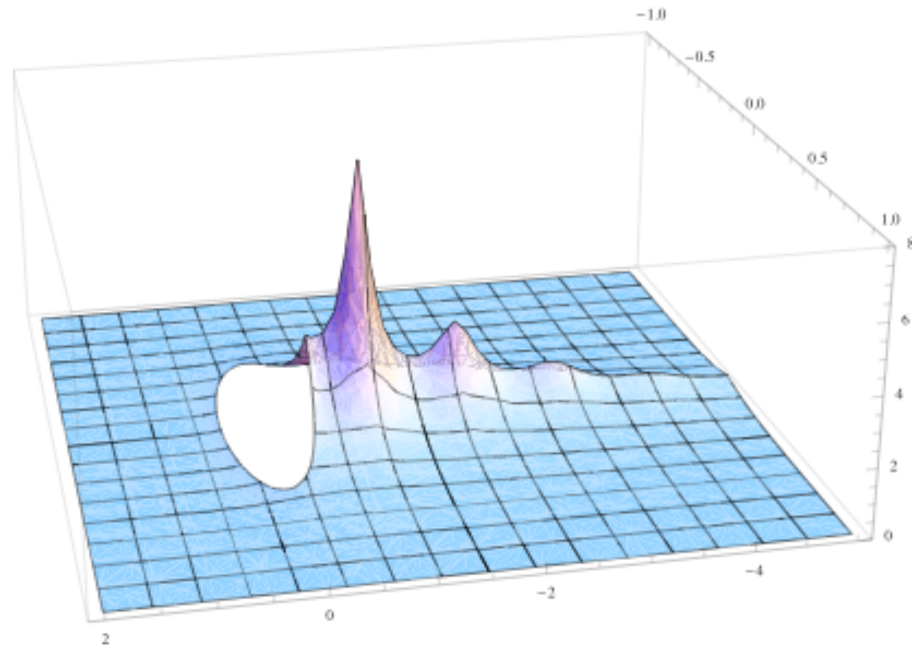
Model for $\Pi(q^2)$ with such behavior does exist! For example:

$$\Pi(z) \;\propto\; \psi(z) = \frac{d\log\Gamma(z)}{dz} = -\sum_{n=0}^{\infty}\frac{1}{z+n} + \mathrm{const}$$

$$z = (q^2)^{\zeta}$$

For $0 < \zeta < 1$, analytic except for cut on (negative) real $q^2$ axis






imaginary part and absolute value of $\Delta(q^2) = \Pi(q^2) - \Pi_{OPE}(q^2)$

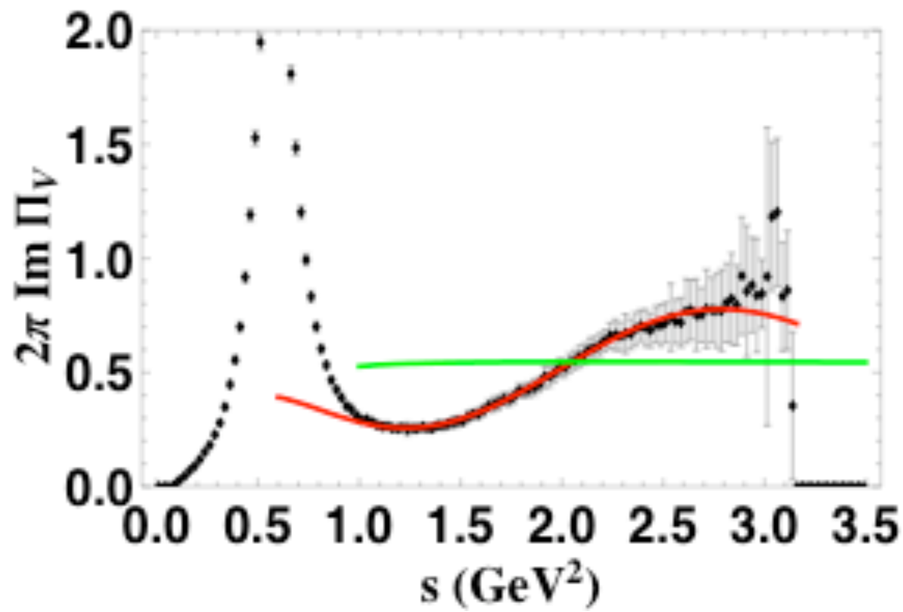
asymptotic behavior along real axis has the form of the *ansatz* ,
if $\gamma = 2\pi^2(1-\zeta) \propto 1/N_c$ small $\rightarrow$ we will try fit

$$\frac{1}{\pi}\,\Pi(s) = \theta(s - s_{min})\left(\frac{N_c}{12\pi^2}[1 + \hat{\rho}_{pert}(s)] + \kappa\, e^{-\gamma s}\ \sin(\alpha + \beta s)\right)$$

directly to spectral functions

# Vector channel:

green: no DVs
red : with DVs



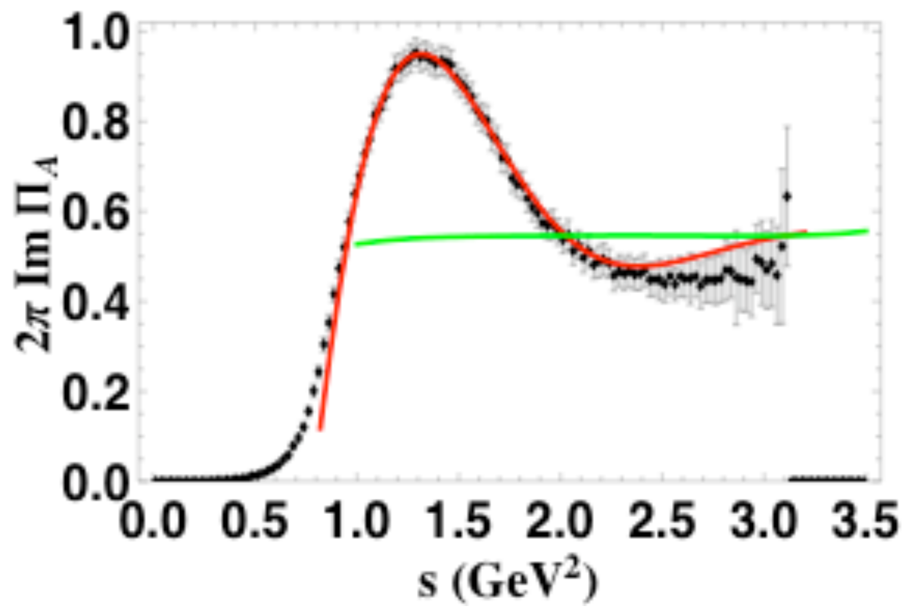

$$\begin{aligned} \kappa &= 0.018(4) \\ \gamma &= 0.15(15)\ \mathrm{GeV}^{-2} \\ \alpha &= 2.2(3) \\ \beta &= 2.0(1)\ \mathrm{GeV}^{-2} \end{aligned}$$

$$\begin{aligned} s_{min} &= 1.1\ \mathrm{GeV}^2 \\ \chi^2/dof &= 10/79 \end{aligned}$$

- FO or CI: no difference
- $\chi^2$ increases for $s$ < 1.1 GeV$^2$, flat above
- central values independent of $s_{min}$

## Axial channel:

green: no DVs
red : with DVs



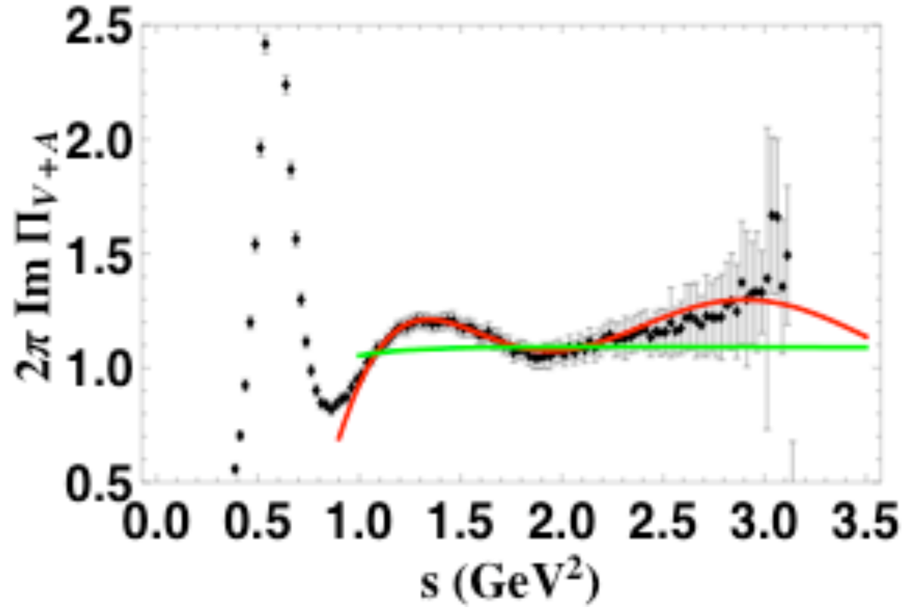

$$\begin{aligned}
\kappa &= 0.20(6) \\
\gamma &= 1.7(2)\ \mathrm{GeV}^{-2} \\
\alpha &= -0.4(1) \\
\beta &= -3.0(1)\ \mathrm{GeV}^{-2}
\end{aligned}$$

$$\begin{aligned}
s_{min} &= 1.1\ \mathrm{GeV} \\
\chi^2/dof &= 17/78
\end{aligned}$$

- FO or CI: no difference
- $\chi^2$ increases for $s$ < 1.1 GeV$^2$, flat above
- central values independent of $s_{min}$

# Vector + axial:

green: no DVs
red : with DVs

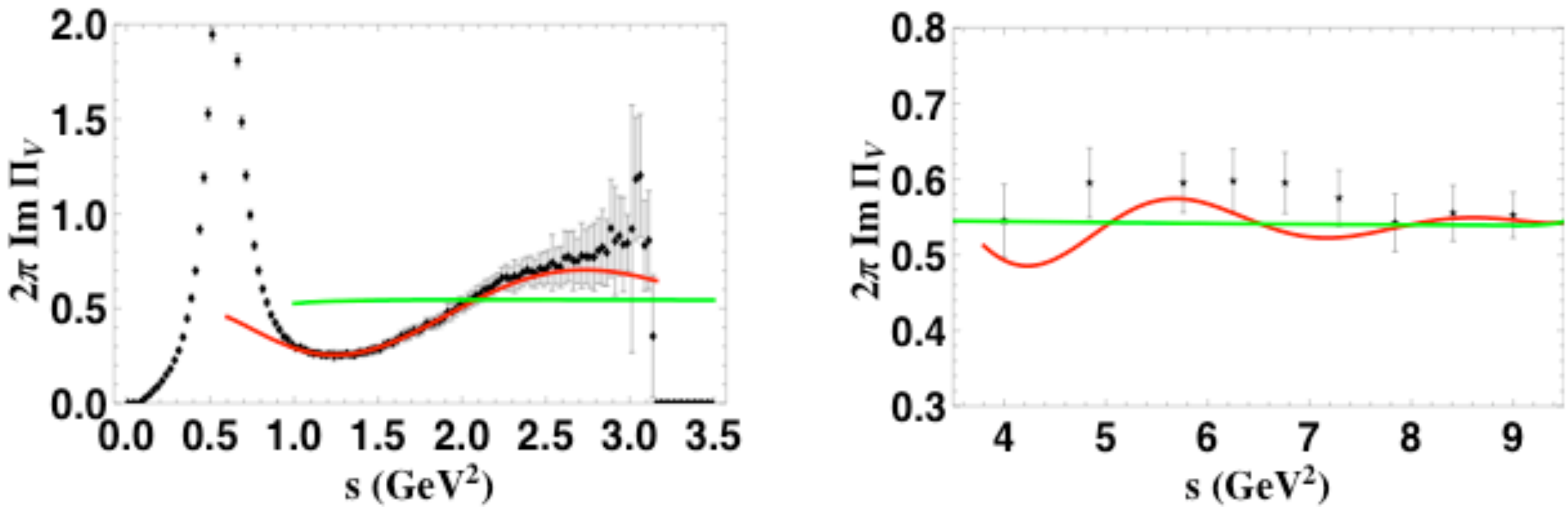



Good description of V+A as well, leads to

$$\delta R_\tau^{V+A} = \delta R_\tau^V = (-0.0022 \pm 0.013)\ S_{EW}|V_{ud}|^2$$
$$\Rightarrow \quad \delta\alpha_s(m_\tau) \sim 0.003 - 0.010$$

compare with theoretical errors 0.005 - 0.015

## Try include $e^+e^-$ data (vector only):




- use only inclusive BES data with $4 < s < 9$ GeV$^2$
- ignore isospin breaking and OZI contributions
- introduce extra parameter $\alpha'$ to account for strange-quark part

$$\kappa = 0.024(4) \qquad \gamma = 0.40(12)$$
$$\alpha = 1.82(19) \qquad \beta = 2.14(11)$$
$$\alpha' = 5.2 \pm 1.4 \qquad \chi^2/dof = 22/87$$

hence

$$\delta\alpha_s(m_\tau) \sim 0.001 - 0.003$$

# Conclusions

- $\alpha_s$ from $\tau$ decays is very precise, but (non)perturbative aspects need to be understood better:
  - higher orders in pert. theory (Baikov et al. (2008), Beneke & Jamin, 2008)
  - higher orders in the OPE (Maltman & Yavin, 2008)
  - duality violations -- no systematic theory

- at present, $\tau$ data do not exclude significant effects from DVs
  - our results differ from Davier et al. (2008)
  - no reason that V+A should be better than V alone
  - $e^+e^-$ may help: DVs suppressed?

- cannot ignore DVs; difficult to get rid of them
  - can better weights be found?
  - can $e^+e^-$ and $\tau$ spectral functions be combined to get a handle?
  - vary models (not optimistic)

- at present:
  effect from DVs comparable to other theoretical errors not ruled out