Gauge Theories, Prepotentials and Loops.

Manu Mathur

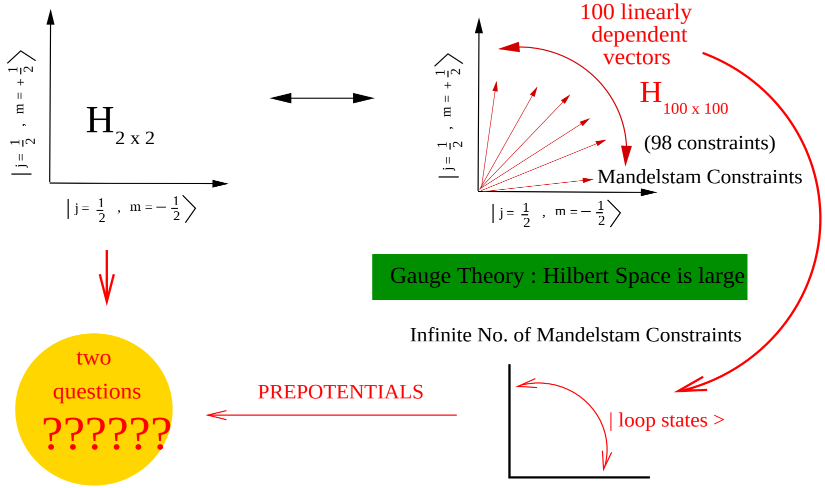
Framework:

1) Gauge Group = SU(2) and SU(3), Space dim. = d,

2) Lattice Hamiltonian Formulation, No matter.

Motivation: Trivial Example

Single Ising Spin at a single lattice site



SU(2) Matrices

U_A : SU(2) Matrix, $\Longrightarrow$ $U_A \equiv A_0 I + i \sum_{a=1}^{3} A_a \sigma^a$,

U_B : SU(2) Matrix, $\Longrightarrow$ $U_B \equiv B_0 I + i \sum_{a=1}^{3} B_a \sigma^a$

$$Tr(U_A) Tr(U_B) = (2A_0)(2B_0) = 4A_0 B_0.$$

$$Tr(U_A\, U_B\,) = (2A_0 B_0 - 2\vec{A}.\vec{B})$$
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This implies:

$$Tr(U_A) Tr(U_B) \equiv Tr(U_A U_B) + Tr(U_A U_B^{-1})$$

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The Mandelstam Constraints

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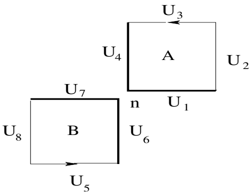


$U_A = U_1 U_2 U_3 U_4$

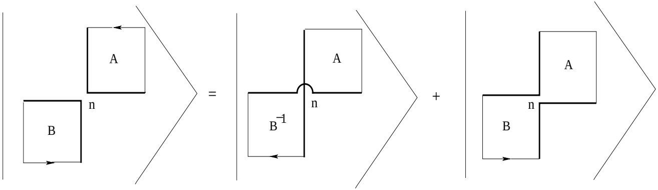
$U_B = U_5 U_6 U_7 U_8$

$|0>$

Strong Coupling

Vacuum

$$Tr(U_A)Tr(U_B)|0\rangle \equiv Tr(U_A U_B^{-1})|0\rangle \quad + \quad Tr(U_A U_B)|0\rangle$$



$$|LS\,1\rangle \quad = \quad |LS\,2\rangle \quad + \quad |LS\,3\rangle$$

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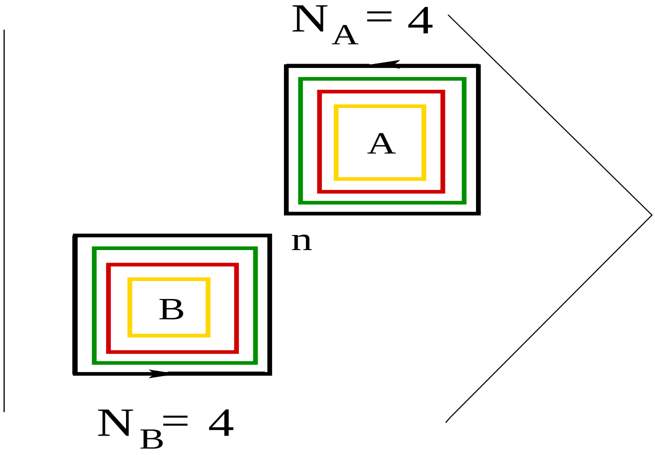

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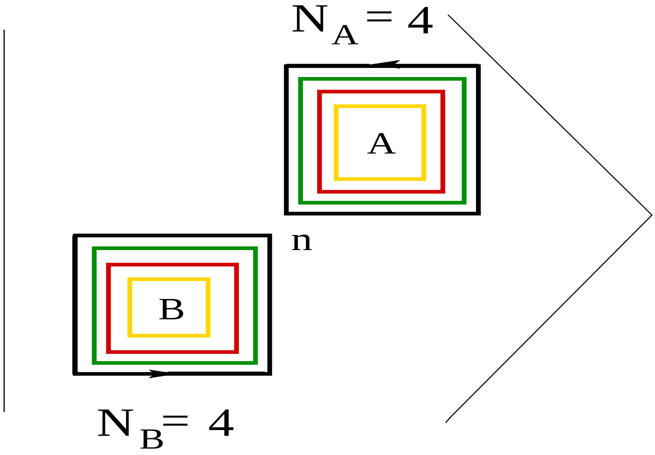

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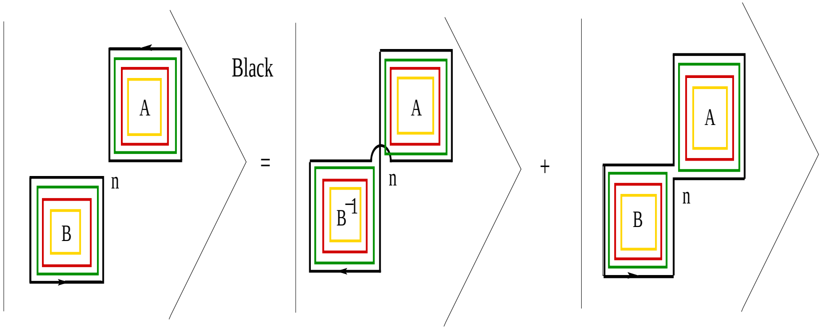

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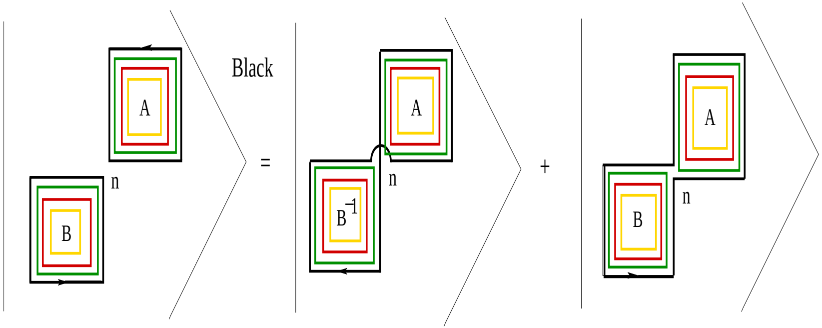

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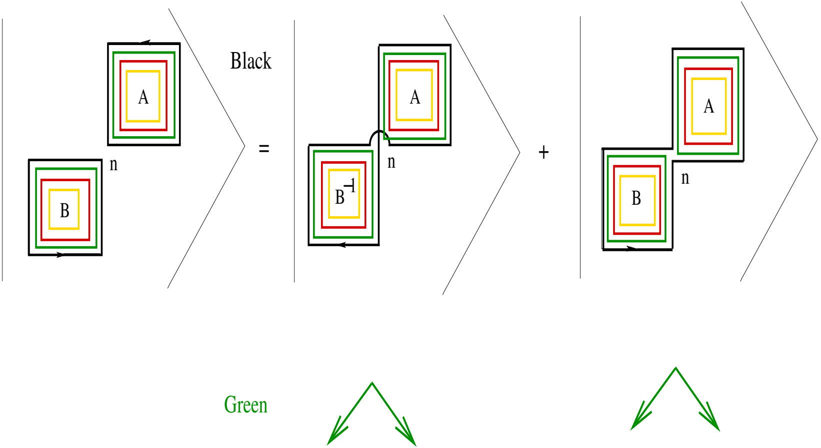

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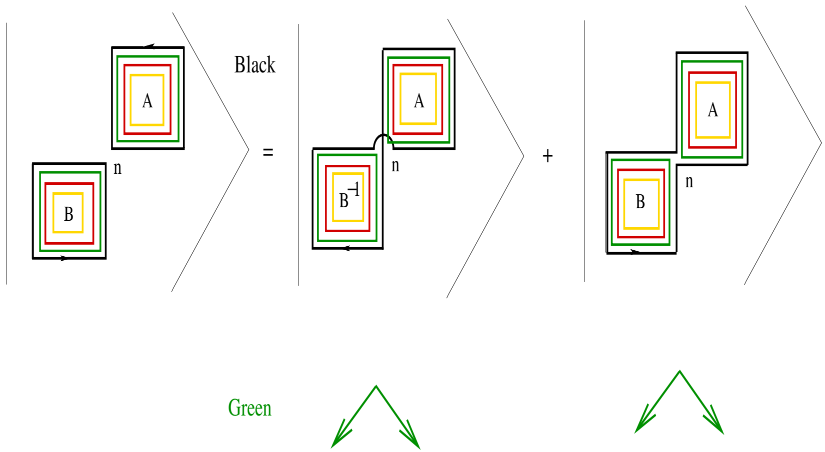

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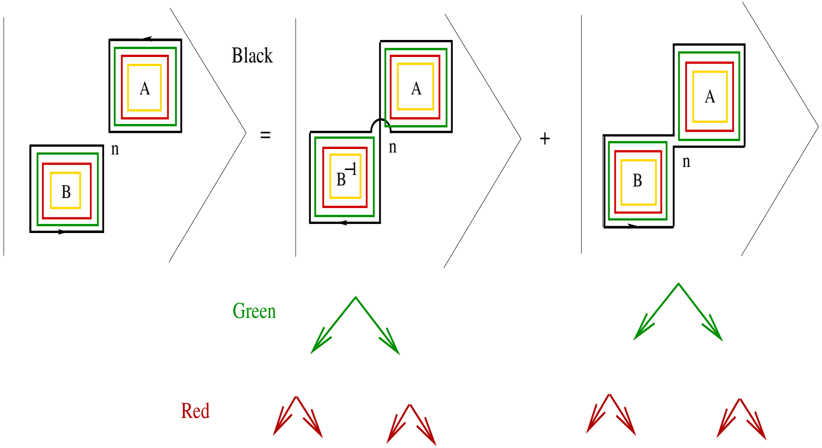

OPEN THE YELLOW LOOPS

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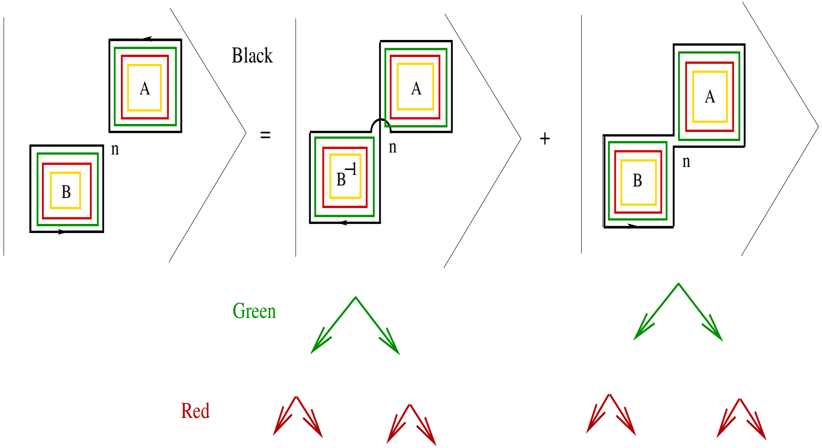

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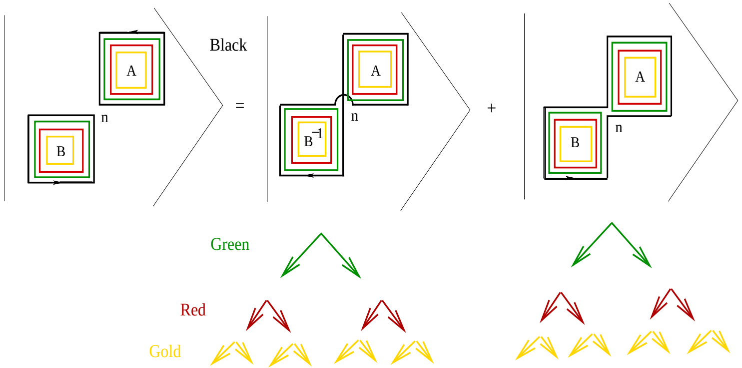

Above is the simplest case.

1. Consider $N_A(=4) \to \infty$, $N_B(=4) \to \infty$! **Large fluxes**. !
2. Consider $A \to A_1 A_2 ...,$ $B \to B_1 B_2 ...$!! **long loops**!!.
3. Consider Many (not just two) loops !!!
4. $d = 2 \to d = 3 \to d$!!!!**arbitrary spatial dimensions.** !!!!

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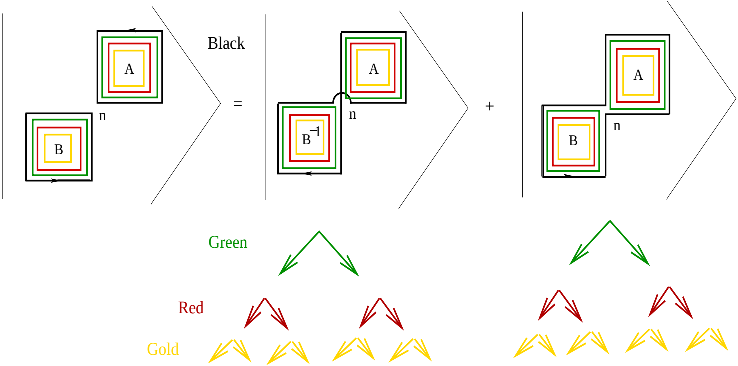

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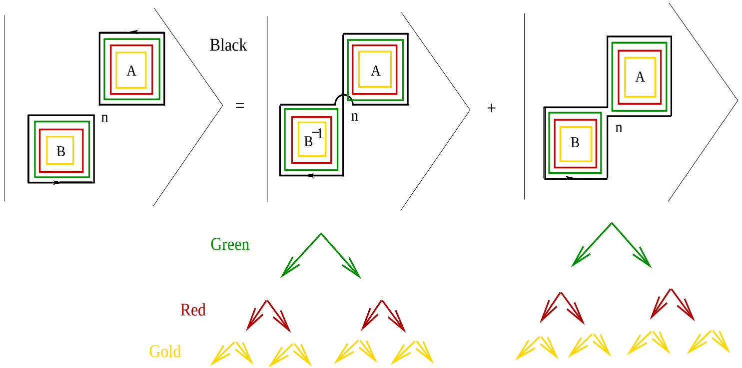

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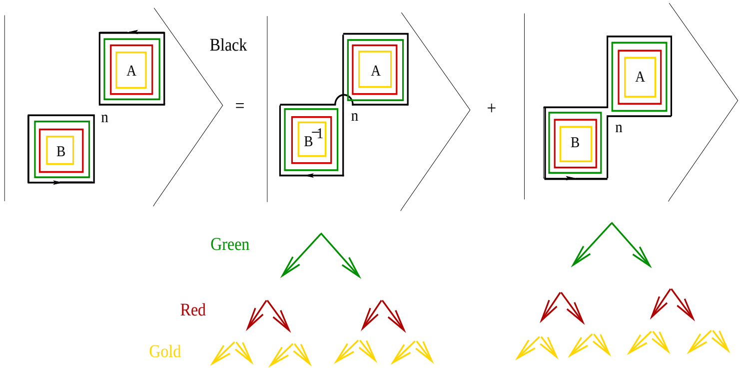

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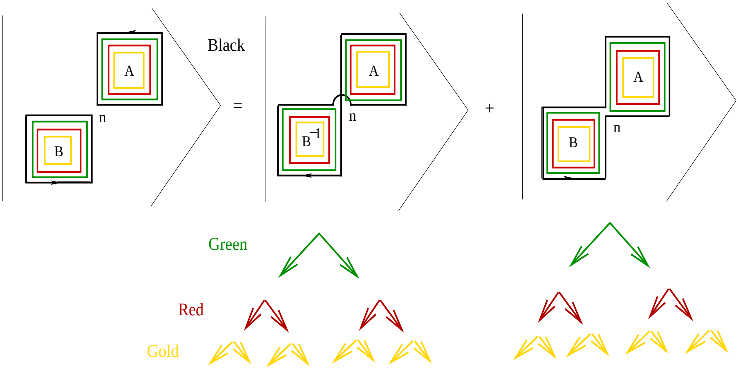

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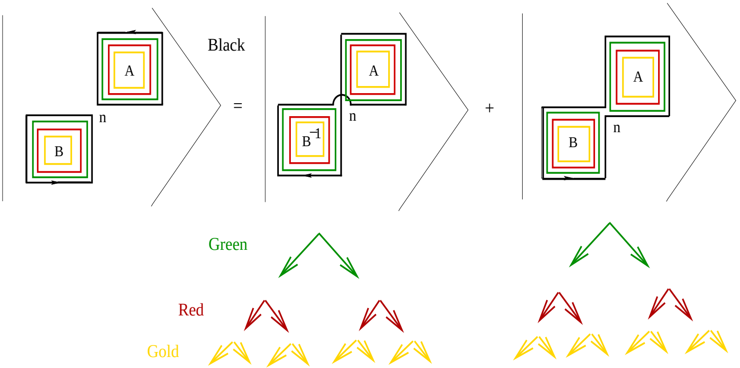

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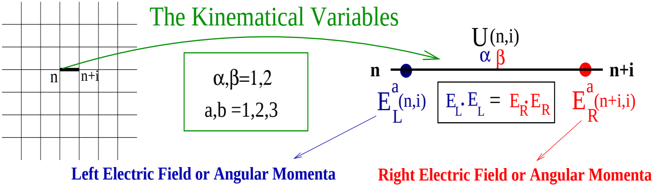
LATTICE GAUGE THEORY : KINEMATICAL VARIABLES

The Kinematical Variables



$\alpha,\beta = 1,2$

$a,b = 1,2,3$

SU(2)

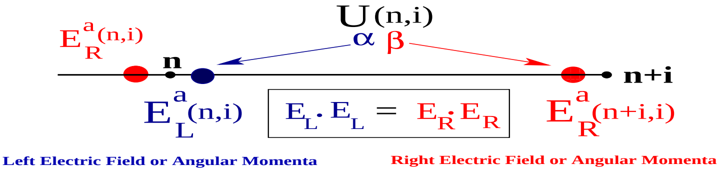
QUANTIZATION

$$[E^a_L(n,i), U_{\alpha\beta}(n,i)] = \sigma^a_{\alpha\bar{\alpha}} U_{\bar{\alpha}\beta}(n,i) \qquad [E^a_R(n+i,i), U_{\alpha\beta}(n,i)] = -U_{\alpha\bar{\beta}}(n,i)\,\sigma^a_{\bar{\beta}\beta}$$

$$[E^a_L(n,i), E^b_L(n,i)] = i\epsilon_{abc}E^c_L(n,i) \qquad [E^a_R(n+i,i), E^b_R(n+i,i)] = i\epsilon_{abc}E^c_R(n+i,i)$$

$$=> E_L(n,i)\cdot E_L(n,i) = E_R(n+i,i)\cdot E_R(n+i,i).$$

$$\left[SU(3) : \alpha,\beta = 1,2,3. \quad a,b,c = 1,2,...,8. \quad \sigma^a \to \lambda^a\right]$$



Gauge Transformations

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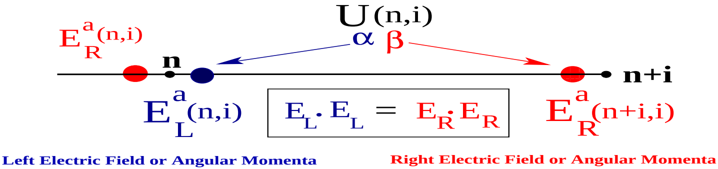

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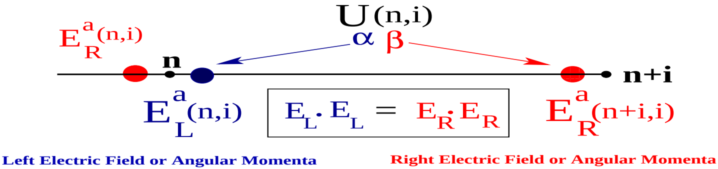

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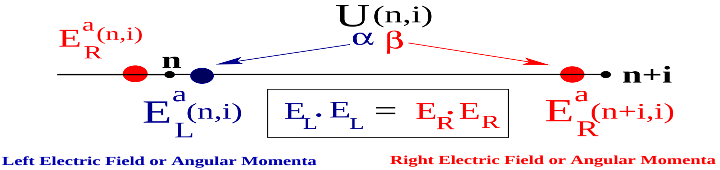

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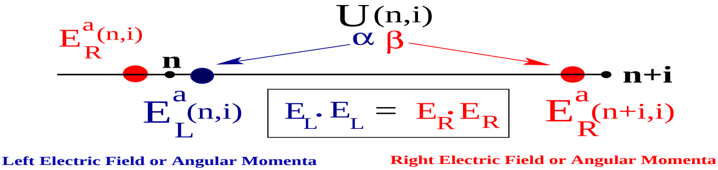

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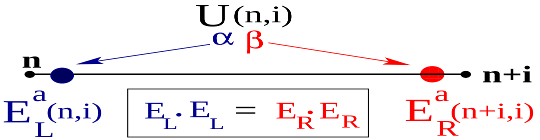

Left Electric Field or Angular Momenta

Right Electric Field or Angular Momenta

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$$=> E^a_L \equiv \frac{1}{2}a^\dagger\sigma^a a, \qquad E^a_R \equiv \frac{1}{2}b^\dagger\sigma^a b.$$

$$E_L \cdot E_L = E_R \cdot E_R$$

Left SU(2) Casimir = **Right SU(2) Casimir**

$$a^\dagger \cdot a = b^\dagger \cdot b \equiv N$$

# of Left Oscillators = **# of Right Oscillators**

$$U_{\alpha\beta} = \frac{1}{\sqrt{N+1}}\left(a_\alpha \tilde{b}_\beta - \tilde{a}^\dagger_\alpha b^\dagger_\beta\right)\frac{1}{\sqrt{N+1}}$$

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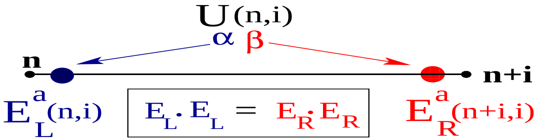

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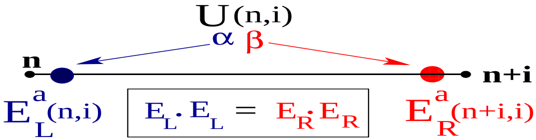

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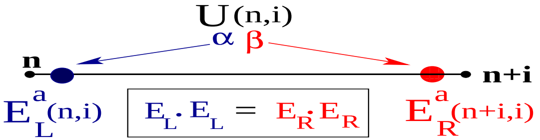

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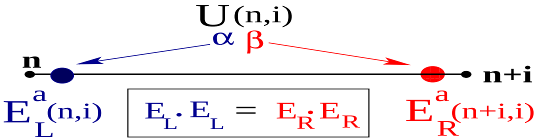

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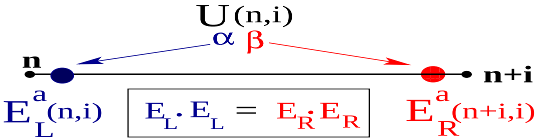

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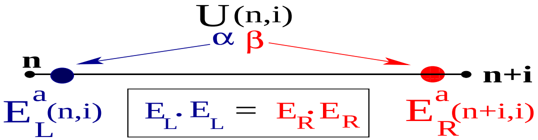


Left Electric Field or Angular Momenta

Right Electric Field or Angular Momenta

$$\left[E^a_L, E^b_L\right] = i\epsilon^{abc}E^c_L, \qquad \left[E^a_R, E^b_R\right] = i\epsilon^{abc}E^c_R.$$

$$=> E^a_L \equiv \frac{1}{2}a^\dagger\sigma^a a, \qquad E^a_R \equiv \frac{1}{2}b^\dagger\sigma^a b.$$

$$E_L \cdot E_L = E_R \cdot E_R$$

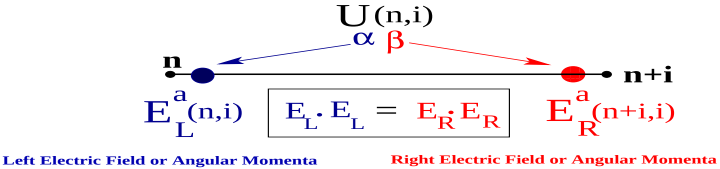
Left SU(2) Casimir = Right SU(2) Casimir

$$a^\dagger \cdot a = b^\dagger \cdot b \equiv N$$

# of Left Oscillators = # of Right Oscillators

$$U_{\alpha\beta} = \frac{1}{\sqrt{N+1}}\left(a_\alpha \tilde{b}_\beta - \tilde{a}^\dagger_\alpha b^\dagger_\beta\right)\frac{1}{\sqrt{N+1}}$$

New Variables: SU(2) Prepotentials / SU(2) Schwinger Bosons



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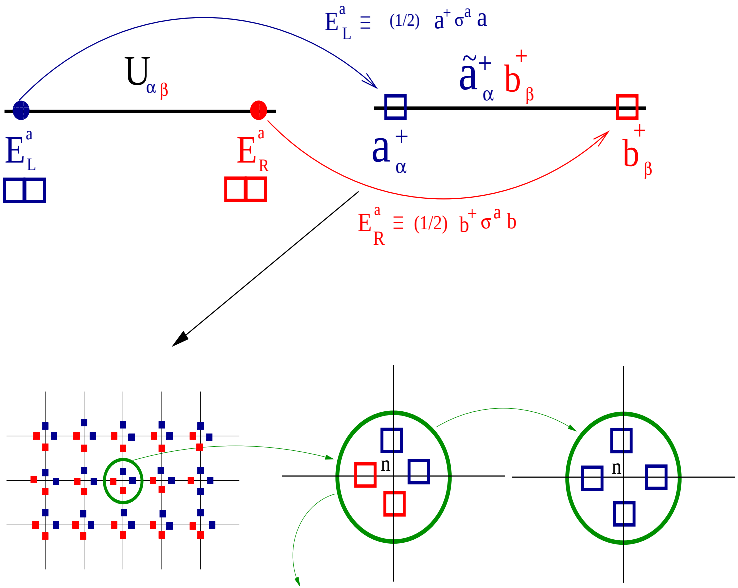
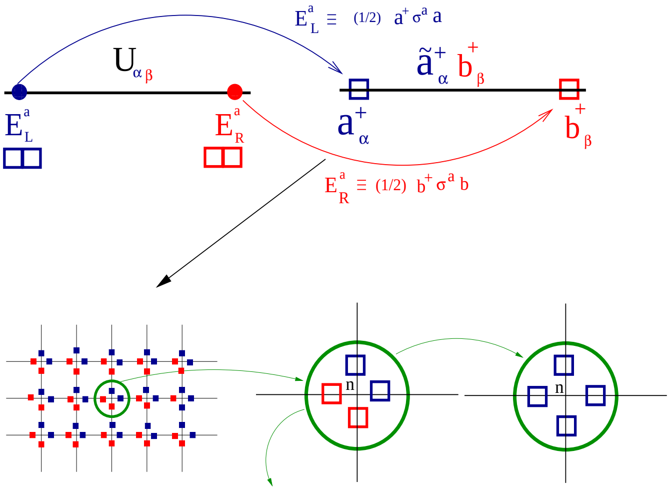
$$U_{\alpha\beta} = \frac{1}{\sqrt{N+1}}\left(a_\alpha \tilde{b}_\beta - \tilde{a}^\dagger_\alpha b^\dagger_\beta\right)\frac{1}{\sqrt{N+1}}$$

The Old Vs. The New SU(2) Language



Four SU(2) doublets around every lattice site (n).

The Old Vs. The New SU(2) Language



Four SU(2) doublets around every lattice site (n).

$E_L \cdot E_L = E_R\ E_R \Rightarrow a^+ \cdot a = a^+ \cdot a$

=> Total no. of left oscillators = Total no. of right oscillators

(U(1) Gauge Invariance.)

The Solutions SU(2): **space dimension =2**

$$|LS\rangle_n \equiv |j_1, j_2, j_3, j_4, j_{12}\rangle = N(j) \sum_{l_{13} l_{14} l_{23} l_{24} l_{34}}{}' \prod_{\substack{i,j \\ i<j}} \frac{1}{l_{ij}!} \left(L_{ij}(n)\right)^{l_{ij}(n)} |0\rangle$$

U(1) Gauge Invariance => 5-d(=2) =3 Loop Coordinates at every lattice site. This construction in terms of U is extremely complicated.

space dimension =d

$$|LS\rangle_n \equiv |j_1, j_2, ..j_{2d}; j_{12}, j_{123}, ...j_{12..(2d-1)} = j_{2d}\rangle$$

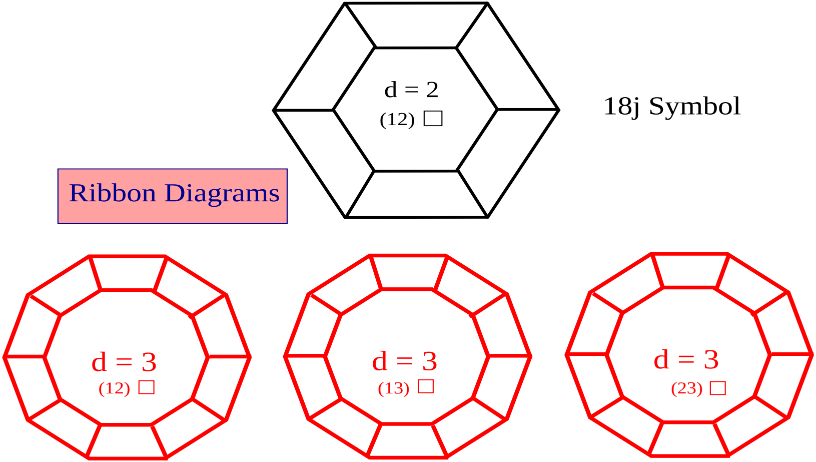
$$= N(j) \sum_{\{l\}}{}' \prod_{\substack{i,j \\ i<j}} \frac{1}{l_{ij}!} \left(L_{ij}(n)\right)^{l_{ij}(n)} |0\rangle$$

U(1) Gauge Invariance => (4d - 3) - d =3 (d - 1) Loop Coordinates at every lattice site.

$\underbrace{\textit{H.S. Sharatchandra and R. Anishetty}, \textit{H. S. Sharatchandra}}_{\textit{duality}}$

The Loop Dynamics

The Matrix Elements of the Magnetic Field Operator



18j Symbol

30j Symbol

SU(3) Prepotentials, (Ramesh Anishetty, Indrakshi Raychowdhury)

$\underbrace{SU(2)}_{\alpha=1,2}$: n — $a^{\dagger\alpha} \in 2$ — (n,i) — $b^{\dagger\alpha} \in 2$ — n+i

$$\mathbf{a}^\dagger \cdot \mathbf{a} = \mathbf{b}^\dagger \cdot \mathbf{b} \equiv n$$

$\underbrace{SU(3)}_{\alpha=1,2,3.}$: n — $a^{\dagger\alpha} \in 3$, $c^\dagger_\alpha \in 3^*$ — (n,i) — $b^{\dagger\alpha} \in 3$, $d^\dagger_\alpha \in 3^*$ — n+i

$$\mathbf{a}^\dagger \cdot \mathbf{a} = \mathbf{b}^\dagger \cdot \mathbf{b} \equiv n$$

$$\mathbf{c}^\dagger \cdot \mathbf{c} = \mathbf{d}^\dagger \cdot \mathbf{d} \equiv m$$

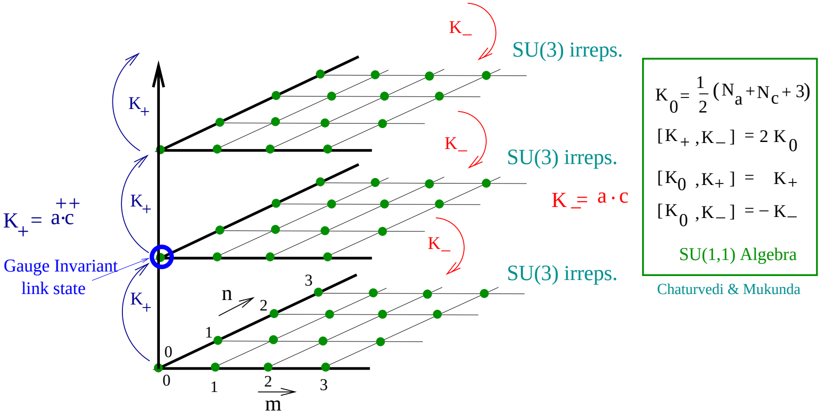
Problem: We also have SU(3) gauge invariant states on every link:

$$(\mathbf{a}^\dagger \cdot \mathbf{c}^\dagger)(\mathbf{b}^\dagger \cdot \mathbf{d}^\dagger)|\mathbf{0}\rangle$$

$\equiv$ Old Problem in Representation theory of SU(N) ($N \geq 3$)
: Problem of multiplicity.

The Multiplicity Problem

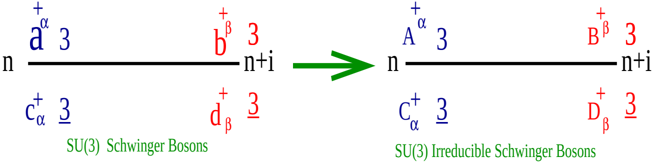
Let $|\psi\rangle^{\alpha_1\alpha_2\ldots\alpha_n}_{\beta_1\beta_2\ldots\beta_m} \in (n, m)$ of SU(3) with $n, m = 0, 1, 2, ..\infty$.



We can show that:

$$\left[K_-, U_\alpha^{\ \beta}\right] = 0.$$

This implies SU(3) Gauge Theory dynamics takes place on the ground floor.



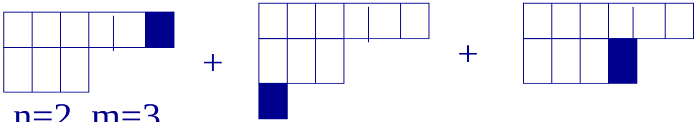
$$\left[\mathbf{K}_-, \mathbf{A}^{\dagger\alpha}\right] = \mathbf{0}, \quad \left[\mathbf{K}_-, \mathbf{C}^{\dagger}_{\alpha}\right] = \mathbf{0}.$$

$$A^{\dagger\alpha} = a^{\dagger\alpha} - f(n_a, n_c)\,\overbrace{(a^{\dagger}\cdot c^{\dagger})}^{K_+}\,c^{\alpha} \in 3 \ (n \rightarrow n+1, m \rightarrow m)$$

$$C^{\dagger}_{\alpha} = c^{\dagger}_{\alpha} - f(n_a, n_c)(a^{\dagger}\cdot c^{\dagger})a_{\alpha} \in 3^{*} \ (n \rightarrow n, m \rightarrow m+1)$$

$$f(n_a, n_c) = \frac{1}{n_a + n_c + 1}$$

$$U_{\alpha}{}^{\beta} = F_1 A^{\dagger}{}_{\alpha} B^{\dagger\beta} + F_2 C_{\alpha} D^{\beta} + F_3 (A \wedge C^{\dagger})_{\alpha} (B \wedge D^{\dagger})^{\beta}$$



n=2, m=3