Nature of QCD-Strings

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Historical Remarks

- With the proliferation of particle species it soon became clear that techniques of Quantum Field Theory that were so efficient in QED were no longer adequate.
- Regularities were soon found among particle properties. It was found that the so called Chew-Frautschi plots of J vs M^2 yielded parallel lines later called Regge Trajectories.
- Shortly afterwards Veneziano proposed a simple formula for scatteing which incorporated the Regge Trajectories.
- Development of Dual Resonance models.

Two Proposals

- It is interesting that at around the same time two radically different proposals came to be made as theories of hadrons.
- One was the astonishing proposal that all hadrons are states of excitation of a Relativistic String.
- The other, more conventional, was a Relativistic Quantum Field Theory of Quarks interacting via the analogue of Photons called Gluons.

Hadronic Strings

- Though the Hadronic String theory accounted naturally for the Regge trajectories as well as the Veneziano formula, they soon ran into severe difficulties.
- One of these was that consistency with Quantum Mechanics and Lorentz invariance required space-time dimensions to be 26.
- The other difficulty was that the spectrum of string theories included massless spin-2 particles which were not hadrons.
- String theories were soon abandoned as candidates for theories of hadrons. Instead,they were proposed as candidates for unification of all known forces including Gravitation.

- The Quark model proposed around the same time was seen by many as only a book-keeping device for the multitude of particles.
- This model was elaborated into a full Relativistic Quantum Field Theory.
- An essential new ingredient was the so called Colour quantum number of quarks.
- Colour is really a representation of the non-abelian group $SU(3)$.
- The analogue of photons were the gluons which carried the 8 dimensional adjoint representation.

Problem of Quark Confinement

- But QCD too ran into a very difficult problem; that of Quark Confinement.
- It was not clear how Absolute Confinement could be a physically meaningful idea.
- The idea of Dual Superconducting Mechanism was proposed as a way of realising absolute confinement.
- One of the consequences of dual superconductivity would be the formation of a thin vortex-like flux tube.
- Does QCD really predict such flux tubes and what are their properties?

Lattice Gauge Theories

- The Quark Confinement problem is very hard because QCD is a non-linear theory with non-abelian gauge invariance.
- The infrared regime of this theory has to be solved non-perturbatively.
- An important development was that of Lattice Gauge Theories.
- The Euclideanised path integral of QCD is restricted to a discrete set of lattice points.
- The problem is identical to one of Classical Statistical Mechanics although in one extra dimension.
- Use computers to simulate the theory.

Studying Flux Tubes in Lattice Gauge Theory.

- Flux tubes can be investigated in LGT by studying configurations with a static pair of quark and anti-quark. Technically this is done by measuring average values of Wilson Loops or Polyakov Loop Correlators.

$$\langle W\rangle \simeq e^{-TV(R)};\ \langle P(0)P^{*}(R)\rangle \simeq e^{-TV(R)}$$

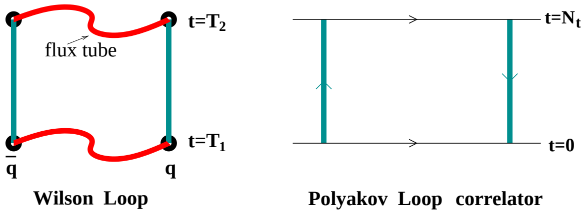

Figure: Wilson Loop and Polyakov Loop Correlators.

Numerical Evidence for Flux Tubes

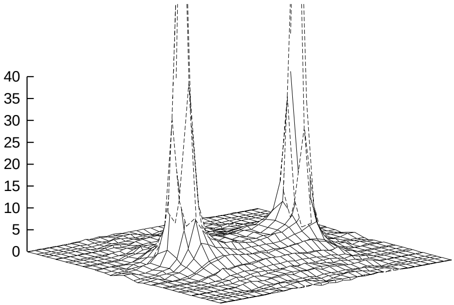

Figure: Static potential between quark-antiquark pair– G. Bali, hep-lat/9409005.

Flux Tubes and Strings

- In 1979 Nambu suggested some formal equivalence between Wilson Loops and Strings.
- Luscher and Weisz developed their effective string description.
- They showed that there is a universal long distance correction to the confining linear potential.

$$V(R) = \sigma R - \frac{(D-2)\pi}{24}\frac{1}{R}$$

- This additional term should be distinguished from the non-universal Coulomb term at short distances which depends on the details of the Gauge group, representation etc.

Numerical Search for the Luscher Term.

- In the early eighties Ambjorn et al and de Forcrand et al reported having measured the Luscher term in numerical simulations.
- These measurements were not very accurate.
- The Luscher term itself is not a clear indicator of the type of string theory involved.
- Dietz and Filk had showed that a variety of theories with different boundary conditions all produce the Luscher term.
- All effective string theories of the type to be discussed shortly also give the Luscher term.
- Naik had shown that AdS/CFT correspondence also yields the Luscher term.
- Non-bosonic strings were essentially ruled out by Lucini et al.

Numerical Evidence for String-like behaviour in QCD

- One measures the Polyakov Loop Correlation functions as these project most accurately onto the ground state of the $q\bar{q}$ system when the temporal extent of the lattice is large enough.

$$K(x,x';T) = \sum_n e^{-E_n\, T}\psi_n(x)\psi_n^*(x')$$

- On the lattice Polyakov loops have the largest possible temporal extent.
- The expectaion values are very small for separations of interest.
- They are around 10^{-26} when the separation is a little over 1 fermi.
- With increase in separation by a lattice unit these expectation values drop by couple of orders.
- One needs specialised algorithms like the Luscher-Weisz Multilevel algorithm.

Luscher-Weisz Multilevel Algorithm.

- This is based on the concept of Transfer Matrices in statistical mechanics.
- It works as long as interactions are local in the sense that their range is a finite number of lattice units.
- In this method one divides the lattice into seceral boundary hypersurfaces and their interriors.
- One updates all the interrior degrees of freedom several times keeping the boundary variables fixed. One computes averages of functions of interrior variables.
- One then multiplies these averages and one finally averages over different boundary values.

Luscher-Weisz Multilevel Algorithm.

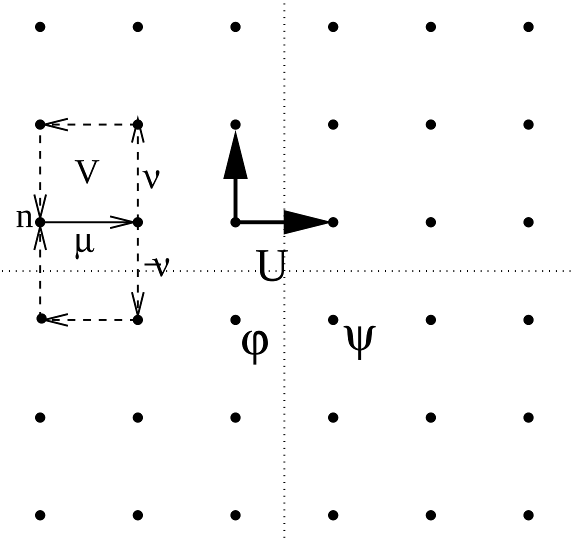

Figure: Subdivision of the lattice for Multilevel algorithm.

Luscher-Weisz Multilevel Algorithm

$$\langle O_1 O_N \rangle = \frac{1}{Z} \int \{dU_l\}\{dU_B\} e^{-S(U_l, U_B)} O_1 \; O_N$$

- For local interactions this can be rewritten as

$$= \frac{1}{Z} \int \{dU_l'\}\{dU_B\} e^{-S(U_l', U_B)} \langle O_1 \rangle_{B_1} \; \langle O_N \rangle_{B_N}$$

  Thus

$$\langle O_1 \; O_N \rangle = \langle \; \langle O_1 \rangle_{B_1} \; \langle O_N \rangle_{B_N} \; \rangle_{bound}$$

- We could speed up simulations by also using analytic multihit method.

Initial Results and Interpretations.

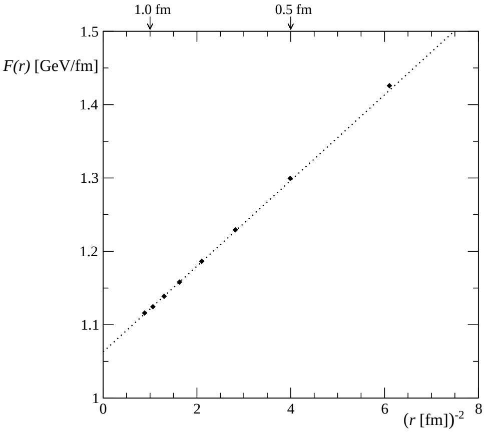

Figure: Force between quark-antiquark pair: LW

A convenient observable.

It is desirable to remove the dominant contribution coming from the linearly rising term in the potential. To this end we define the scaled second derivative

$$c(r) = -\frac{r^3}{2}V''(r)$$

For large distances this is expected to approach the Luscher term $\frac{(D-2)\pi}{24}$. Deviations from this give information about subleading terms in the potential.

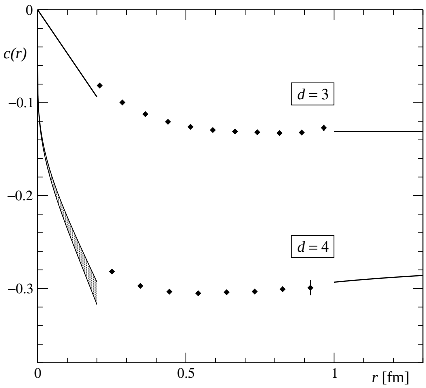

Figure: Scaled second derivative of the potential: LW

Boundary Terms?

Initially LW thought that the difference of about 15% could be ascribed to boundary terms in the action in addition to the free bosonic string action.

$$S = \frac{1}{2T}\int d\sigma d\tau \partial X \cdot \partial X$$

of the form

$$\frac{b}{4}\int d\tau \partial_\sigma X \cdot \partial_\sigma X|_{bound}$$

These give additional contributions to the potential of the form

$$\Delta V(r) = \frac{b}{r^2}$$

Open-Close String Duality

However in 2004 they showed that open-close string duality forbids such boundary terms and so $b = 0$.

Our Simulations

- With Pushan Majumdar I have carried out extensive simulations in both $d = 3$ and $d = 4$.
- In $d = 3$ we have studied $SU(2)$ on much larger lattices at various lattice spacings.
- In $d = 3$ we have covered a large range of distances.
- In $d = 4$ we have studied $SU(3)$ on much larger lattices than LW and to much larger separations.
- I show the results in the next few slides.

Our Results for $d = 4$ SU(3): c(r)

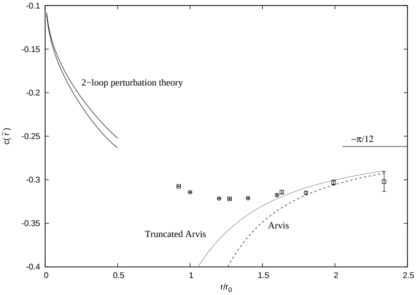

Figure: Scaled second derivative of the potential for $d = 4$

Our Results for $d = 3$: Force.

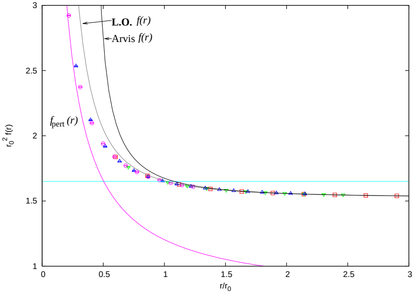

Figure: Our results for force in 3d SU(2)

Our Results for $d = 3$: c(r).

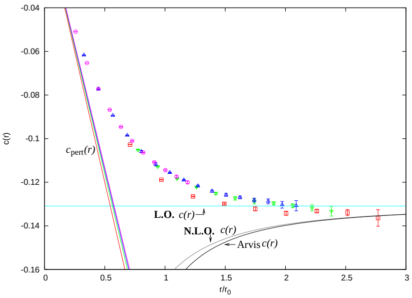

Figure: Scaled second derivative of the $d = 3$ potential

Evidence for Bosonic Strings in QCD

- What these high accuracy simulations reveal is that the best fit for the potential expanded as

$$V(r) = \sigma r + \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} + \cdot$$

  is one which agrees with the values of c_i given by Bosonic String theory.
- The exact potential cominmg from Bosonic string theory is(Arvis)

$$V(r) = \sqrt{\sigma^2 r^2 - \frac{(d-2)\pi}{12}}$$

- Thus for large distances what superficially looks like the one gluon coulombic $\frac{1}{r}$ is not coulombic at all as it is *universal*.
- It depends only on d not on charges or even the gauge group! This is the well-known Luscher Term.

Does this make sense?

- It is immediately obvious that the string discovered through numerical simulations of QCD can not be the fundamental Bosonic String!
- The fundamental bosonic string lives in 26 dimensions while QCD is a fully consistent theory in 4 dimensions!
- The bosonic string suffers from tachyonic instability, whereas the ground state of the static quark-antiquark sector should be stable and show no such instability..

String-like defects

- String-like defects are common in physics. Some of the well known ones are vortices in superfluids, fluxtubes in superconductors, vortices in BE condensates, Nielsen-Olesen vortices of field theories, cosmic strings, flux tubes in QCD etc..
- Depending upon physical parameters these objects could behave quantum mechanically and the challenge is to find consistent quantum descriptions of such objects.
- If one were to follow quantisation schemes used in String Theory, one immediately runs into severe problems.
- For Bosonic String theories, consistent quantisations are possible only in 26 dimensions.
- For supersymmetric strings this critical dimension gets lowered to 10.

Effective Theories

- Polchinski and Strominger suggested to treat such string-like defects by an Effective String Theory
- The idea is very similar to how we treat the low energy dynamics of Pions and Nucleons. Though in principle QCD should be able to provide a description of this sector, it is in the nonperturbative sector.
- Instead one uses the idea of an Effective Field Theory wherein one tries to retain the symmetry features of QCD like chiral symmetry but otherwise put no restrictions like renormalisability etc. on the type of allowed actions.

Chiral Symmetric Effective Theories

- The chiral effective action

$$\mathcal{L}_\pi = \frac{1}{2}\mathcal{D}_\mu\vec{\pi}\mathcal{D}^\mu\vec{\pi}$$

  for example, is very succesful in describing the low energy interactions among the pions.
- But this action is obviously not renormalisable.
- Nevertheless loop effects can be calculated as systematic expansions in derivatives. The lack of renormalisability manifests through the appearance of arbitrary constants at each order.
- But at each order these are a few in number which can be determined phenomenologically.

Polchinski-Strominger Effective String Theories

- The PS prescription is to write all action terms that are invariant under conformal transformations.
- Drop all terms proportional to the leading order constraints $\partial_\pm X \cdot \partial_\pm X = 0$.
- Drop all terms proportional to the leading order equations of motion $\partial_{+-} X^\mu = 0$.
- Leading order is in the following sense:

$$X^\mu_{\rm cl} = e^\mu_+ R\tau^+ + e^\mu_- R\tau^-; \tag{1}$$

where $e^2_- = e^2_+ = 0$ and $e_+ \cdot e_- = -1/2$ satisfies the full EOM.

- Fluctuations around the classical solution are denoted by Y^μ, so that

$$X^\mu = X^\mu_{\rm cl} + Y^\mu. \tag{2}$$

Polchinski-Strominger Action

- $$S = \frac{1}{4\pi}\int d\tau^+ d\tau^- \left\{ \frac{1}{a^2}\partial_+ X^\mu \partial_- X_\mu + \beta \frac{\partial_+^2 X \cdot \partial_- X \partial_+ X \cdot \partial_-^2 X}{(\partial_+ X \cdot \partial_- X)^2} + \mathcal{O}(R^{-3}) \right\}. \quad (3)$$

- This action is invariant, i.e $\delta S < \mathcal{O}(R^{-2})$, under the modified conformal transformations

$$\delta_- X^\mu = \epsilon^-(\tau^-)\partial_- X^\mu - \frac{\beta a^2}{2}\partial_-^2 \epsilon^-(\tau^-)\frac{\partial_+ X^\mu}{\partial_+ X \cdot \partial_- X}, \quad (4)$$

- (and another; $\delta_+ X$ with $+$ and $-$ interchanged).

Motivation from Liouville Theory

- In $D \neq 26$ the conformal factor in the $d = 2$ metric does not decouple, instead its induced action is

$$S_{Liou} = \frac{26 - D}{48\pi} \int d\tau^+ \, d\tau^- \;\; \partial_+\phi\partial_-\phi \qquad (5)$$

- Replace the conformal factor e^ϕ by the component $\partial_+X \cdot \partial_-X$ of the induced metric on the world sheet.
- Replace $(26 - D)/12$ by a parameter β to be determined by requiring the vanishing of the total central charge in all dimensions.

$$\begin{aligned} S_{(2)} &= \frac{1}{4\pi} \int \mathrm{d}\tau^+ \mathrm{d}\tau^- \left\{ \frac{1}{a^2} \partial_+X^\mu \partial_-X_\mu \right. \\ &\left. + \beta \frac{\partial_+(\partial_+X \cdot \partial_-X)\partial_-(\partial_+X \cdot \partial_-X)}{(\partial_+X \cdot \partial_-X)^2} \right\}. \end{aligned} \qquad (6)$$

- Dropping leading order EOM just leads to PS action.

Consistency in all dimensions.

- The energy-momentum tensor in terms of the fluctuation field is

$$T_{--} = -\frac{R}{a^2} e.\partial_- Y - \frac{1}{2a^2}\partial_- Y \cdot \partial_- Y - \frac{\beta}{R} e_+ \cdot \partial_-^3 Y + \ldots \quad (7)$$

- The Operator Product Expansion(OPE) of $T_{--}(\tau^-)T_{--}(0)$ is given by

$$\frac{D + 12\beta}{2(\tau^-)^4} + \frac{2}{(\tau^-)^2} T_{--} + \frac{1}{\tau^-}\partial_- T_{--} + \mathcal{O}(R^{-1}). \quad (8)$$

Consistency in all dimensions.

- In the absence of the PS action the OPE would have given D as the matter central charge.
- In order to cancel the central charge -26 from gauge fixing D would have to be 26.
- But now the special value $\beta = \beta_c$ with $\beta_c = \frac{26-D}{12}$ cancels the gauge fixing central charge for all values of D!

Spectrum of PS Theory.

- Making the mode expansion $\partial_- Y^\mu = a \sum_{m=-\infty}^{\infty} \alpha_m^\mu e^{-im\tau^-}$
- The Virasoro generators are given by

$$\begin{aligned} L_n &= \frac{R}{a} e_- \cdot \alpha_n + \frac{1}{2} \sum_{m=-\infty}^{\infty} : \alpha_{n-m} \cdot \alpha_m : \\ &+ \frac{\beta_c}{2} \delta_n - \frac{a \beta_c n^2}{R} e_+ \cdot \alpha_n + \mathcal{O}(R^{-2}). \end{aligned} \tag{9}$$

- The quantum ground state is $|k, k; 0\rangle$ which is also an eigenstate of α_0^μ and $\tilde{\alpha}_0^\mu$ with common eigenvalue ak^μ. This state is annihilated by all α_n^μ for positive-definite n.

Spectrum of PS Theory.

- The ground state momentum is

$$p^{\mu}_{\text{gnd}} = \frac{R}{2a^2}(e^{\mu}_{+} + e^{\mu}_{-}) + k^{\mu}$$

- The total rest energy is

$$(-p^2)^{1/2} = \sqrt{\left(\frac{R}{2a^2}\right)^2 - k^2 - \frac{R}{a^2}(e_+ + e_-) \cdot k}. \qquad (10)$$

- The physical state conditions $L_0 = \tilde{L}_0 = 1$ fix k, so that

$$k^1 = 0, \qquad k^2 + \frac{R}{a^2}(e_+ + e_-) \cdot k = \frac{(2 - \beta_c)}{a^2}. \qquad (11)$$

Spectrum of PS Theory.

$$(-p^2)^{1/2} = \frac{R}{2a^2}\sqrt{1 - \frac{D-2}{12}\left(\frac{2a}{R}\right)^2}, \tag{12}$$

- This is the precise analog of the result obtained by Arvis for open strings.
- Expanding this and keeping only the first correction, one obtains for the static potential

$$V(r) = \frac{R}{2a^2} - \frac{D-2}{12}\frac{1}{R} + \cdots . \tag{13}$$

Absence of R^{-3} corrections to the Arvis spectrum.

- Peter Matlock and I, and Drummond, have shown that there is no correction to the Nambu-Goto spectrum at the R^{-3} level also!
- This requires showing first that there are no candidate action terms whose leading behaviour is R^{-3}.
- It follows that though L_n get corrected, L_0 does not.
- There are subtle issues of measure, quantum equivalence of field theories etc.
- Our proof of absence of corrections at R^{-3} respects all these requirements.
- At the level of classical equivalence of field theories Drummond has shown that there are no candidate action terms at R^{-4}, R^{-5} level also.
- This does not immediately imply that there are no corrections to the spectrum at these orders.

Some more recent results

- Peter Matlock and myself have recently finished developing a covariant calculus for a systematic construction of effective string theories.
- Some surprises: the integrand of the PS action can not be covariantised!
- PS action owed its origin to the Liouville action whose origin is the WZNW action for Weyl anomaly in two dimensions(first derived by Polyakov)
- Integrands of such actions can not be written in general covariant manner!
- Our analysis also clarified the true symmetry content of string theories.

Investigating Higher Order Terms in $V(R)$

- Both from the standpoint of a theoretical understanding as well as comparison with numerical simulations, it is very important to calculate higher order terms at least for some particular effective string theories.
- Our covariant formalism provides a systematic approach for these investigations.
- That formalism gives a completion of the PS term which is valid for all orders in R^{-1}.
- Peter Matlock and myself are currently calculating the R^{-4}, R^{-5} corrections to the spectrum.

Thick Strings

- The flux tubes of QCD have thickness. Hence it is very important to incorporate thickness into the effective description.
- At this stage it is not clear how to provide an ab initio description of thick strings.
- A starting point based on membrane theories could be technically etremely hard.
- An interesting idea in this regard is that of Polchinski and Susskind, who have argued that the four dimensional projection of certain thin AdS_5 behave like thick strings. I am investigating this line of thought with Vikram Vyas.
- In the AdS approach, integrating out the radial coordinate would result in an effective description in four dimensions. It will be interesting to see the class of effective effective string theories that emerge.

Future Directions

- At what order do corrections to the Nambu-Goto spectrum really enter? With the dependence on D changed to (D-26) a new, larger, scale enters the picture.
- Are the deviations from Nambu-Goto spectrum seen in simulations due to these potential corrections?
- Or do they indicate a breakdown of the string picture itself?
- At intermediate distances there has to be a dependence on the gauge group as the data at shorter distances is in good agreement with predictions of asymptotic freedom which certainly is sensitive to the gauge group.
- Do such nonuniversal terms still have an interpretation in effective string theory?

Future Directions

- Is it possible to derive the effective string picture starting from QCD, at least in the large N limit?
- How does the tachyon of bosonic strings gets cured?
- What is the connection of all this to the conjectured duality between strings and gauge theories?
- AdS-CFT correspondence also gives the correct Luscher term as shown by S. Naik. It will be interesting to calculate the R^{-3} terms from this angle.
- Extrinsic curvature: are extrinsic curvature terms as conjectured by Polyakov important? Or by the time they manifest is their no effective string at all? At first sight it may appear that the extrinsic curvature action, being proportional to the leading order equation of motion, can be transformed away by field redefinitions. But as carefully emphasized by us, the resulting change in measure must be carefully taken into account.

Future Directions

- Effect of dynamical quarks.
- As simulations with fermions are so expensive it may be possible to learn about dynamical colour matter fields by simulating with scalar quarks. This is the SU(3) non-abelian Gauge-Higgs system.
- Finite temperature studies: do the strings survive past the deconfining temperature?
- Supersymmetric generalisations.

A short bibliography

- Joseph Polchinski and Andrew Strominger, *Effective String Theory*, Phys.Rev.Lett **67**,1681(1991).
- N.D. Hari Dass and Pushan Majumdar, *String-like behaviour of 4-D Yang-Mills flux tubes*, JHEP 0610:020,2006.
- N.D. Hari Dass and Peter Matlock, *Universality of correction to Luscher term in Polchinski-Strominger effective string theories*,`hep-th/0606265`.
- J.M. Drummond, *Reply to `hep-th/0606265`*,`hep-th/0608109v1`.
- N.D. Hari Dass and Peter Matlock, *Our response to the response `hep-th/0608109` by Drummond*, `hep-th/0611215`.

More bibliography

- N.D. Hari Dass and Peter Matlock, *Field Definitions, Spectrum and Universality in Effective String Theories*, submitted to JHEP.
- N.D. Hari Dass and Pushan Majumdar, *Continuum Limit of String Formation in 3-d SU(2) LGT*, `hep-lat/0702019`, to be published in Physics Letts B.
- N.D. Hari Dass and Pushan Majumdar, *Subleading properties of the QCD fluxtube in 3-d SU(2) LGT*, Proceedings of Lattice 2007.
- N.D. Hari Dass and Peter Matlock, *A Covariant Calculus for Effective String Theories*, .
- J. Polchinski and L. Susskind, *String Theory and the Size of Hadrons*,`hep-th/0112204v2`.