Non-perturbative regularization of QCD

P. Mitra

Saha Institute of Nuclear Physics, Calcutta

Abstract

On the one hand the CP violating theta term of QCD can be converted to a phase in the quark mass term. On the other hand, QCD with a complex mass term for quarks can be regularized so as not to violate CP. There is a crucial dependence on the regularization or measure. The symmetries of some regularization schemes are discussed and implications for CP pointed out.

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Introduction

Interaction	Symmetry	Quark currents
Electromagnetic	P, T	$\frac{2}{3}\bar{q}\gamma_\mu q - \frac{1}{3}\bar{\tilde{q}}\gamma_\mu \tilde{q}$
Strong	P, T	$\bar{q}\frac{\lambda^i}{2}\gamma_\mu q + \bar{\tilde{q}}\frac{\lambda^i}{2}\gamma_\mu \tilde{q}$
W^+	$\not{P}$, T	$\bar{q}_L\gamma_\mu \tilde{q}_L$
W^-	$\not{P}$, T	$\bar{\tilde{q}}_L\gamma_\mu q_L$
Z	$\not{P}$, T	$\bar{q}_L\gamma_\mu q_L - \bar{\tilde{q}}_L\gamma_\mu \tilde{q}_L -$ $2\sin^2\theta_W(\frac{2}{3}\bar{q}\gamma_\mu q - \frac{1}{3}\bar{\tilde{q}}\gamma_\mu \tilde{q})$
Gravitation	P, T	$\bar{q}\frac{\sigma_{mn}}{2}\gamma_l e^l_\mu q + \bar{\tilde{q}}\frac{\sigma_{mn}}{2}\gamma_l e^l_\mu \tilde{q}$

- q denotes quarks with charge $\frac{2}{3}$ and $\tilde{q}$ with charge $-\frac{1}{3}$
- Spontaneous symmetry breaking: vacuum expectation value of Higgs: Mass terms $\bar{q}_L M q_R + \bar{\tilde{q}}_L \tilde{M} \tilde{q}_R$+hc with complex matrices arise from symmetry breaking through Yukawa interactions.

INTRODUCTION

- On diagonalization of $M, \tilde{M}$,

$$q_L \to A_L^{-1} q_L, \qquad q_R \to A_R^{-1} q_R$$
$$\tilde{q}_L \to \tilde{A}_L^{-1} \tilde{q}_L, \qquad \tilde{q}_R \to \tilde{A}_R^{-1} \tilde{q}_R,$$

  only W-interactions pick up matrix $A_L \tilde{A}_L^{-1} \equiv C$, the [Cabibbo]-Kobayashi-Maskawa matrix, may be complex

- Some phases of C may be absorbed in quark phases, by multiplying $A_L, \tilde{A}_L$ by suitable diagonal matrices; surviving ones, if any, $\Rightarrow \not{T}$ in weak interactions – experimentally observed

- Kobayashi and Maskawa showed that for three generations, one such phase remains (though not for two generations)

- Diagonalized mass terms may also be complex $\Rightarrow \not{P}, \not{T}$ in strong interactions?

INTRODUCTION

- $\bar{\psi}_L m e^{i\theta'} \psi_R + hc = \bar{\psi} m e^{i\theta'\gamma_5} \psi$
  $$= \cos\theta' \bar{\psi} m \psi + i \sin\theta' \bar{\psi} m \gamma_5 \psi.$$
  Looks like scalar + pseudoscalar
  $\Rightarrow$ suggests parity violation
- Phase factor $e^{i\theta'} \to e^{-i\theta'}$ under antilinear operation
  $\Rightarrow$ suggests time-reversal violation
- A chiral transformation can be used to generate the phase $e^{i\theta'\gamma_5}$ in the mass term and conversely its inverse
  $$\psi \to e^{-i\theta'\gamma_5/2}\psi, \quad \bar{\psi} \to \bar{\psi} e^{-i\theta'\gamma_5/2}$$
  may be used to remove θ' from the action ...

INTRODUCTION

... but it reappears in the effective action:

$$\theta' g^2 F\tilde{F}/32\pi^2$$

$\Rightarrow$ P, T violation
This is because of the chiral **anomaly**
A $\theta g^2 F\tilde{F}/32\pi^2$ term may also occur *intrinsically*

- Old estimate of T-violating electric dipole moment of neutron: about $10^{-16}\theta'$ e-cm, to be compared with experimental upper bound of 10^{-26} e-cm. $\Rightarrow \theta' < 10^{-10}$, whereas phases in Kobayashi-Maskawa matrix are of the order of 10^{-5}
- Modifications of QCD proposed to suppress such values
- One approach: use chiral symmetry
  Mass may be generated by Yukawa coupling to special pseudoscalar field which preserves chiral symmetry of action and acquires vacuum expectation value

This produces a Goldstone boson: this (**axion**) however remains undetected

- Another approach: use symmetries to make $\theta' = 0$
- Unfortunately, none of these models has been experimentally verified
- However, θ' does not really lead to P or T violation in classical field theory: they just get redefined. In quantum field theory *the regularization or measure has to be taken into account.*

Under reflection about the vertical,

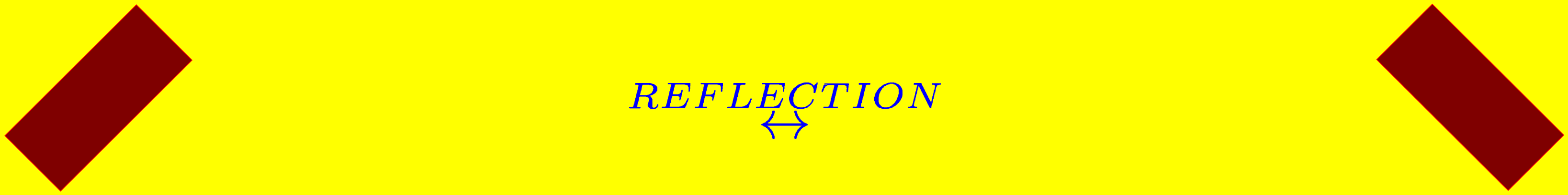
 $\overset{REFLECTION}{\longleftrightarrow}$ 

some things do not change

Under reflection about the vertical,

some things do not change

while some things do:



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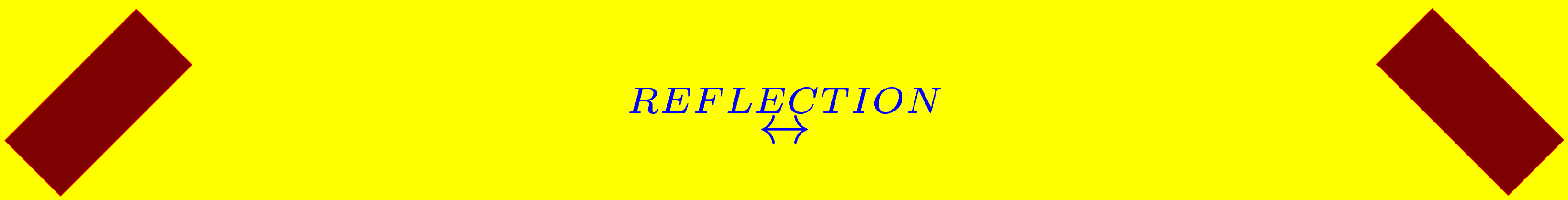

Is the reflection symmetry broken in the second case?

INTRODUCTION

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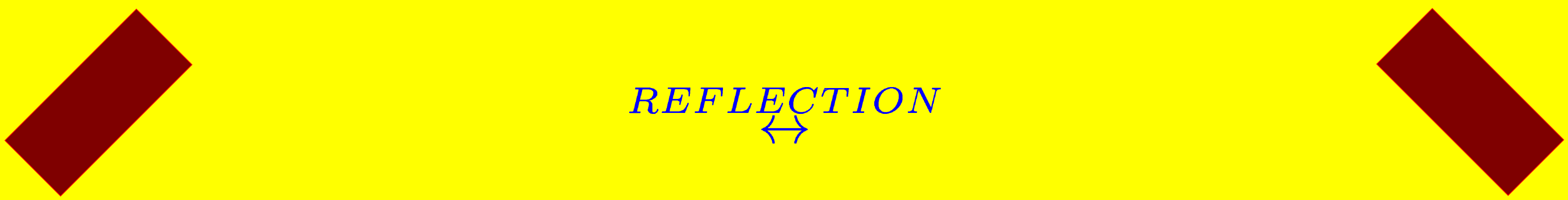

Is the reflection symmetry broken in the second case?

The object is the same, the symmetry must be the same, but it has to be rotated with the object.

In the same way, the symmetries P, T get chirally rotated because of θ' at the classical level.

In quantum field theory, *the regularization or measure has also to be taken into account.*

Construction of Parity for fermions with real mass term

Seek invariance under

$$\psi(\vec{r}) \to P\psi(-\vec{r})$$

of

$$\int \bar{\psi}(i\gamma^\mu D_\mu - m)\psi.$$

$$\Rightarrow P^\dagger P = 1, \quad -P^\dagger\gamma^0\gamma^i P = \gamma^0\gamma^i, \quad P^\dagger\gamma^0 P = \gamma^0$$

$$\Rightarrow -P^\dagger\gamma^0 P P^\dagger\gamma^i P = \gamma^0\gamma^i$$

$$\Rightarrow -P^\dagger\gamma^i P = \gamma^i$$

Satisfied by $P = \gamma^0$

$$\begin{aligned}\mathcal{P}\bar{\psi}\psi\mathcal{P}^{-1} &= \bar{\psi}(-\vec{r})\psi(-\vec{r}) \\ \mathcal{P}\bar{\psi}\gamma_5\psi\mathcal{P}^{-1} &= -\bar{\psi}(-\vec{r})\gamma_5\psi(-\vec{r})\end{aligned}$$

CONSTRUCTION OF PARITY

For complex mass term, the situation changes:
Seek invariance under

$$\psi(\vec{r}) \to P\psi(-\vec{r})$$

of

$$\int \bar{\psi}(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi.$$

$$\Rightarrow P^\dagger P = 1, \quad -P^\dagger\gamma^0\gamma^i P = \gamma^0\gamma^i, \quad P^\dagger\gamma^0 e^{i\theta'\gamma_5}P = \gamma^0 e^{i\theta'\gamma_5}$$

$$\Rightarrow -P^\dagger\gamma^0 e^{i\theta'\gamma_5}PP^\dagger\gamma^i e^{i\theta'\gamma_5}P = \gamma^0 e^{i\theta'\gamma_5}\gamma^i e^{i\theta'\gamma_5}$$

$$\Rightarrow -P^\dagger\gamma^i e^{i\theta'\gamma_5}P = \gamma^i e^{i\theta'\gamma_5}$$

Satisfied by $P = \gamma^0 e^{i\theta'\gamma_5} = e^{-i\theta'\gamma_5/2}\gamma^0 e^{i\theta'\gamma_5/2}$

$$\begin{aligned}\mathcal{P}\bar{\psi}e^{i\theta'\gamma_5}\psi\mathcal{P}^{-1} &= \bar{\psi}(-\vec{r})e^{i\theta'\gamma_5}\psi(-\vec{r}) \\ \mathcal{P}\bar{\psi}\gamma_5 e^{i\theta'\gamma_5}\psi\mathcal{P}^{-1} &= -\bar{\psi}(-\vec{r})\gamma_5 e^{i\theta'\gamma_5}\psi(-\vec{r})\end{aligned}$$

Construction of Time-reversal: real mass term

Seek invariance under the antilinear operation

$$\psi(x^0) \to T\psi(-x^0)$$

of

$$\int \bar{\psi}(i\gamma^\mu D_\mu - m)\psi.$$

$$\Rightarrow T^\dagger T = 1, \quad -T^\dagger \gamma^{0*}\gamma^{i*} T = \gamma^0\gamma^i, \quad T^\dagger \gamma^{0*} T = \gamma^0$$

$$\Rightarrow -T^\dagger \gamma^{0*} T T^\dagger \gamma^{i*} T = \gamma^0 \gamma^i$$

$$\Rightarrow -T^\dagger \gamma^{i*} T = \gamma^i$$

In standard representation γ^2 purely imaginary, rest real:

$$T^\dagger \gamma^\mu T = \gamma^\mu (\mu = 0, 2), T^\dagger \gamma^\mu T = -\gamma^\mu (\mu = 1, 3)$$

Satisfied by $T = i\gamma^1\gamma^3$

CONSTRUCTION OF TIME − REVERSAL

For complex mass term, the situation changes:
Seek invariance under the antilinear operation

$$\psi(x^0) \to T\psi(-x^0)$$

of

$$\int \bar{\psi}(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5})\psi.$$

$$\Rightarrow T^\dagger T = 1, \quad -T^\dagger \gamma^{0*}\gamma^{i*} T = \gamma^0\gamma^i, \quad T^\dagger \gamma^{0*} e^{-i\theta'\gamma_5^*} T = \gamma^0 e^{i\theta'\gamma_5}$$

$$\Rightarrow -T^\dagger \gamma^{0*} e^{-i\theta'\gamma_5^*} T T^\dagger \gamma^{i*} e^{-i\theta'\gamma_5^*} T = \gamma^0 e^{i\theta'\gamma_5} \gamma^i e^{i\theta'\gamma_5}$$

$$\Rightarrow -T^\dagger \gamma^{i*} e^{-i\theta'\gamma_5^*} T = \gamma^i e^{i\theta'\gamma_5}$$

If γ^μ in standard representation, satisfied by

$$T = e^{i\theta'\gamma_5} T_{standard} = ie^{i\theta'\gamma_5}\gamma^1\gamma^3$$

General class of functional measures

Consider fermion action with real mass term and invariant under parity P:

$$S[\psi,\bar{\psi},A] = S[P\psi,\bar{\psi}P,A^P].$$

Here A is some gauge field and A^P its parity-transformed form.

Note that a chiral transformation $\exp(i\theta'\gamma^5/2)$ on the fields generates the complex mass term containing $\bar{\psi}\exp(i\theta'\gamma^5)\psi$.

More generally, apply a chiral transformation $\chi = \exp(i\beta\gamma^5/2)$. The action will change: no longer invariant under P, but see!

$$\begin{aligned} S_\chi[\psi,\bar{\psi},A] &\equiv S[\chi\psi,\bar{\psi}\chi,A] \\ &= S[P\chi\psi,\bar{\psi}\chi P,A^P] \\ &= S[\chi\chi^{-1}P\chi\psi,\bar{\psi}\chi P\chi^{-1}\chi,A^P] \\ &= S_\chi[\chi^{-1}P\chi\psi,\bar{\psi}\chi P\chi^{-1},A^P]. \end{aligned}$$

This new symmetry simply involves P rotated by χ.

What about fermion regularization or measure of functional integration? For action S, the conventional fermion measure, with implicit regularization, is invariant under P:

$$d\mu[\psi, \bar{\psi}, A] = d\mu[P\psi, \bar{\psi}P, A^P].$$

But this $d\mu$ does not in general respect the rotated parity symmetry as there is a chiral anomaly:

$$d\mu[\psi, \bar{\psi}, A] \neq d\mu[\chi^{-1}P\chi\psi, \bar{\psi}\chi P\chi^{-1}, A^P].$$

Manufacture a new measure!

$$\begin{aligned} d\mu_\chi[\psi, \bar{\psi}, A] &\equiv d\mu[\chi\psi, \bar{\psi}\chi, A] \\ &= d\mu[P\chi\psi, \bar{\psi}\chi P, A^P] \\ &= d\mu_\chi[\chi^{-1}P\chi\psi, \bar{\psi}\chi P\chi^{-1}, A^P], \end{aligned}$$

which formally has the same symmetry as S_χ.

From above construction new measure involves β through χ.

For each phase θ' in mass term, there exists a class of measures parametrized by β.

Note that these constructions are in the presence of gauge fields: anomalies have been implicitly taken into account:
it is only because of anomalies that measure $d\mu_\chi$ depends on χ.

Now, one may take

$$\beta = \theta' :$$

there exists a choice of the measure such that its **symmetry coincides with that of the complex fermion action**.

Parity is then not broken by the combination of the mass term and the measure.
Similar arguments apply for time-reversal.
These may be broken only by a θ term in the action.

$\mathcal{PAULI-VILLARS\ \ REGULARIZATION}$

In generalized Pauli-Villars regularization, Lagrangian density augmented to include extra species:

$$\mathcal{L}_{\psi,\ reg} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\phi}_{jk}(i\gamma^\mu D_\mu - M_j)\phi_{jk}$$

ϕ_{jk}: regulator spinor fields with fermionic or bosonic statistics, with signs $\pm$ for integers c_j satisfying

$$1 + \sum_j c_j = 0, \quad m^2 + \sum_j c_j M_j^2 = 0$$

to cancel divergences

Ultimately, regulators $M_j \to \infty$

PAULI – VILLARS REGULARIZATION

Measure of integration now includes Pauli-Villars fields:

$$d\mu = d\psi d\bar{\psi} \prod_{jk} d\phi_{jk} d\bar{\phi}_{jk}$$

$$Z_{reg} \equiv \int d\mu e^{i \int d^4x \mathcal{L}_{\psi,\ reg}}$$

Consider a chiral phase in mass term: $m \exp(i\theta' \gamma^5)$

A phase β may be introduced in **measure**:

$$d\mu_\chi = d(\chi\psi) d(\bar{\psi}\chi) \prod_{jk} d\phi_{jk} d\bar{\phi}_{jk}$$

where $\chi = \exp(i\beta\gamma^5/2)$. If in particular $\beta = \theta'$,

$$Z^1_{reg} \equiv \int d\mu_\chi e^{i \int d^4x \mathcal{L}_{\chi\psi,\ reg}}$$

is seen to be independent of θ' as $\chi\psi$ may be redefined as ψ'.

$\Rightarrow$ θ'-dependence *removed by choice of measure*

PAULI − VILLARS REGULARIZATION

Alternatively, a phase β may be introduced in the action to provide *a chiral phase in regulator mass terms*:

$$\mathcal{L}_{\psi,\ reg}^{[\theta',\beta]} = \bar{\psi}(i\gamma^\mu D_\mu - m e^{i\theta'\gamma_5})\psi + \sum_j \sum_{k=1}^{|c_j|} \bar{\phi}_{jk}(i\gamma^\mu D_\mu - M_j e^{i\beta\gamma_5})\phi_{jk}$$

In the general case, the measure $d\mu$ may be modified to include both θ' and β.

PAULI – VILLARS REGULARIZATION

In the special case $\beta = \theta'$, it is interesting to use the unchanged measure $d\mu$.

Use chiral transformation to remove θ' from physical fermion as well as regulators:

Jacobian factors corresponding to $\phi_{jk}, \bar{\phi}_{jk}$ come with powers $c_j/|c_j|$ because of fermionic/bosonic statistics, while $\psi, \bar{\psi}$ obey fermionic statistics

$$J_{reg} = e^{-i(1+\sum_j c_j)\int d^4x(\theta'/2)g^2(F\tilde{F}/16\pi^2)} = 1$$

In regularized framework, Jacobian for *combined* chiral transformation on physical fermion fields and regulators is known to be trivial

$$Z_{reg}^{[\theta',\theta']} \equiv \int d\mu \exp\{i \int d^4x \mathcal{L}_{\psi,\ reg}^{[\theta',\theta']}\} = Z_{reg}^{[0,0]}$$

$\Rightarrow \theta'$-dependence *can be removed by choice of measure*

Lattice regularization: Wilson fermions:

$$S = a^4 \sum_x \bar{\psi}[\gamma^\mu \frac{D_\mu + D_\mu^*}{2} - aD_\mu^* D_\mu - me^{i\theta' \gamma_5}]\psi.$$

a= lattice spacing, D_μ= forward covariant difference operator on the lattice, D_μ^*= backward covariant difference operator

Chiral anomaly manifests itself in Wilson formulation as explicit breaking of chiral symmetry of action – no non-trivial Jacobian arises from measure; anomaly means *no* regularization can preserve symmetry

Fermionic part of action *not* invariant under (redefined) parity, with link variables transformed in usual way: parity anomaly?

Can *other* formulations of lattice regularization preserve symmetry?

LATTICE REGULARIZATION

Recall Seiler-Stamatescu regularizations

$$S = a^4 \sum_x \bar{\psi}[\gamma^\mu \frac{D_\mu + D_\mu^*}{2} - aD_\mu^* D_\mu e^{i\beta\gamma_5} - me^{i\theta'\gamma_5}]\psi$$

parametrized by β.

Phase also allowed in actions satisfying Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aD\gamma_5 D$: ($D$= generalized lattice Dirac operator)

$$S = a^4 \sum_x \bar{\psi}[e^{\frac{i\beta\gamma_5}{2}} D e^{\frac{i\beta\gamma_5}{2}} - me^{i\theta'\gamma_5}]\psi,$$

Choose $\beta = \theta'$: redefined parity, with link variables transformed in the usual way, leaves fermionic action invariant![a]

Non-generic regularization, but existence of *one* parity conserving regularization means parity not anomalous. Take $a \to 0$ with $\beta = \theta'$ and the parity continues to be conserved. Same is true of T.

[a]Measure does not break ordinary chiral symmetry even for Ginsparg-Wilson

Zeta function regularization

The formal functional integral of the exponential of a fermion action involving an operator X is

$$\int d\mu \exp[\int \bar{\psi} X \psi] = \det X.$$

The determinant of a matrix can be thought of as the product of its eigenvalues. For an operator, the product of the eigenvalues has to be regularized.

The zeta function of X involves a parameter s,

$$\zeta(s, X) \equiv \mathrm{Tr}(X^{-s}).$$

In terms of eigenvalues λ, this becomes

$$\zeta(s, X) = \sum (\lambda^{-s}),$$

so that

$$\zeta'(s,X) = -\sum(\ln\lambda\lambda^{-s}),$$

and

$$\zeta'(0,X) = -\sum(\ln\lambda) = -\ln\prod\lambda = -\ln\det X.$$

This provides a definition of the determinant. The eigenvalues are assumed to be positive.

When there is a phase in the mass term

$$S_M = -\int\bar{\psi}m\exp(i\theta'\gamma^5)\psi.$$

Note that the Dirac operator

$$i\not\partial + \not A - m\exp(i\theta'\gamma^5)$$

is neither hermitian nor antihermitian.

ZETA FUNCTION REGULARIZATION

A **positive** operator is needed for the zeta function. It is constructed as

$$\Delta = [i \not\partial + \not A - m\exp(i\theta'\gamma^5)]^\dagger [i \not\partial + \not A - m\exp(i\theta'\gamma^5)].$$

For **antihermitian** γ-matrices, (euclidean spacetime), this is *independent* of the phase θ':

$$\Delta = -(i \not\partial + \not A)^2 + m^2.$$

The zeta function of this operator is

$$\zeta(s, \Delta) \equiv \mathrm{Tr}(\Delta^{-s}),$$

and the regularized **logarithm of the fermion functional integral** is defined in the limit of $s \to 0$ as

$$-\frac{1}{2}\zeta'(0, \Delta) - \frac{1}{2}\ln\mu^2\zeta(0, \Delta).$$

A square root is involved because of the squaring in the construction of Δ.

The result is independent of θ', depends on the gauge fields through the operator Δ.

This is the same determinant as in the case where the phase θ' in the mass term vanishes and is therefore invariant under P,T transformations of the gauge field A.

Confirms that a phase in the quark mass term does not cause any P,T violation in quantum chromodynamics if a suitable regularization is used.

GRAVITATIONAL INTERACTION

In a gravitational background, the basic Dirac operator is

$$\not{D} = \gamma^l e_l^\mu (\partial_\mu - iA_\mu - \frac{i}{2} A_\mu^{mn} \sigma_{mn}).$$

Here, e_l^μ: tetrad, A_μ^{mn}: spin connection, and $\sigma_{mn} = i[\gamma_m, \gamma_n]$.

Fermion action (for real mass term) is

$$\int \bar{\psi}(i \not{D} - m)\psi.$$

Invariant under parity transformation

$$\begin{aligned} \psi(\vec{x}) &\rightarrow \gamma^0 \psi(-\vec{x}), \\ e_l^\mu(\vec{x}) &\rightarrow \pm e_l^\mu(-\vec{x}), \end{aligned}$$

where the $\pm$ sign is negative if an odd number of Greek or Latin spatial indices is involved.

Note: σ_{mn} produces similar $\pm$ sign when γ^0 taken across it and cancels $\pm$ sign from Latin spatial indices of A_μ^{mn}.

Invariant under time-reversal transformation

$$\begin{aligned} \psi(x^0) &\rightarrow i\gamma^1\gamma^3\psi(-x^0), \\ e_l^\mu(x^0) &\rightarrow \pm e_l^\mu(-x^0), \end{aligned}$$

where the $\pm$ sign is negative if an odd number of Greek or Latin temporal indices is involved.

Note: σ_{mn} produces similar $\pm$ sign and cancels $\pm$ sign from Latin temporal indices of A_μ^{mn}.

Fermion action for complex mass term is

$$\int \bar{\psi}(i \not{D} - m \exp(i\theta'\gamma^5))\psi.$$

Invariant under chirally rotated parity and time-reversal:

$$\psi(\vec{x}) \rightarrow \gamma^0 \exp(i\theta'\gamma^5)\psi(-\vec{x}).$$

$$\psi(x^0) \rightarrow i \exp(i\theta'\gamma^5)\gamma^1\gamma^3\psi(-x^0).$$

As regards zeta function regularization,

$$\not{D} = \gamma^l e_l^\mu (\partial_\mu - iA_\mu - \frac{i}{2} A_\mu^{mn} \sigma_{mn})$$

for euclidean signature is hermitian in the scalar product

$$(\phi, \psi) \equiv \int d^4x \sqrt{g} \phi^\dagger \psi.$$

The required positive operator is then defined as

$$\begin{aligned} \Delta &= [i \not{D} - m \exp(i\theta' \gamma^5)]^\dagger \\ &\quad [i \not{D} - m \exp(i\theta' \gamma^5)]. \end{aligned}$$

The arguments for the gauge theory can be repeated: θ' does not enter $\Delta = (\not{D})^2 + m^2$ or the fermion determinant which is defined in terms of

$$\zeta(s, \Delta) \equiv \text{Tr}(\Delta^{-s}).$$

CONCLUSION

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- One may adjust the value of any intrinsic *vacuum angle* θ to be zero (or non-zero if necessary in future) in accord with experiments currently negating parity and time-reversal violation in the strong interactions. No non-trivial fine-tuning.

Conclusion

- Parity, time-reversal are redefined in the presence of the chiral phase θ' arising from the mass terms. If the fermion measure is chosen to have the symmetry of the fermion action, no violation because of θ'.
- One may adjust the value of any intrinsic *vacuum angle* θ to be zero (or non-zero if necessary in future) in accord with experiments currently negating parity and time-reversal violation in the strong interactions. No non-trivial fine-tuning.
- This is in standard theory and does not require axions or other hypothetical constructs.