Spin-String Interaction in QCD

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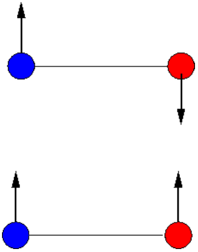
Outline

Based on Vikram Vyas, *Spin-string Interaction in QCD Strings*, Phys. Rev. D78 045003 (2008), arXiv:0704.2707 [hep-th]

Pion and Rho in a String Picture?

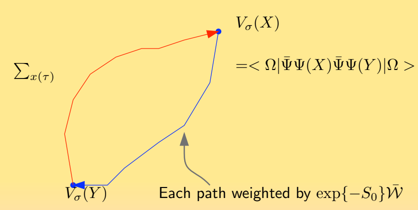
- Can one understand Pion-Rho mass difference?
- Can one understand spontaneous breaking of chiral symmetry in a string description?



"*How do the quark spins couple to the string variables?* " - Polchinski (1990)

Worldline representation of meson propagators

$$\Delta_\sigma(X-Y) = -\int_0^\infty \frac{dT}{T} \exp\left\{-\frac{m^2}{2}T\right\} \int_{x,\psi} \exp\{-S_0[x,\psi]\}$$
$$\times < \mathcal{W} >_{YM} V_\sigma(X,x,\psi) V_\sigma(Y,x,\psi),$$
$$S_0 = \int_0^T d\tau \left\{\frac{\dot{x}^2}{2} + \frac{1}{2}\psi_\mu\dot{\psi}_\mu\right\}$$



$$\psi_\mu \leftrightarrow \gamma_\mu$$
$$\int_\psi \leftrightarrow \mathrm{Tr}_\gamma$$

Wilson Loop for Spin-half Particle (SWL)

$$\mathcal{W}[x(\tau), \psi(\tau)] = \mathrm{Tr}\hat{P}\exp\left\{ i \oint d\tau \left(\frac{dx}{d\tau}.A - \frac{1}{2}\psi_\mu \psi_\nu F_{\mu\nu} \right) \right\},$$

This is often called the super Wilson loop (SWL) as it is invariant under a worldline SUSY

$$\delta x = \epsilon\psi;\ \delta\psi = -\epsilon\dot{x}.$$

Berezin (1976), Brink et el. (1976), Barducci et el. (1976).

Banks-Casher Criterion for SBCS

$$\lim_{m\to 0} V_\chi = m\int_0^\infty dT \exp\left\{-\frac{m^2}{2}T\right\}\int_{y,\psi}\exp\{-S_0\} < \mathcal{W} >_{YM} \neq 0,$$

requires

$$\lim_{T\to\infty}\int_{y,\psi}\exp(-S_0) < \mathcal{W} >_{YM} = \frac{C_0}{T^{1/2}} + \mathcal{O}\left(\frac{1}{T^n}\right)$$

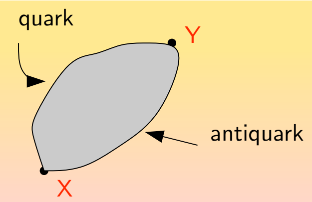
- $\lim_{T\to\infty}\int_{y,\psi}\exp\{-S_0\} \propto \frac{1}{T^{D/2}}$, random walk in D dimensions.
- Banks and Casher (1980): spin-dependent interaction, described by the SWL are responsible for an effective one-dimensional behavior.

Planar QCD as a String Theory: String Representation of SWL?

Can one write

$$< \mathcal{W}[x(\tau), \psi(\tau)] >_{YM} = \int DX(\sigma) \exp\{-S_{SWL}[X, x, \psi]\},$$

where $S[X, x(\tau), \psi(\tau)]$ is the action of an open string whose endpoints lie on the worldline of a quark and an anti-quark. Such an action would contain any spin-string interaction that might exist.



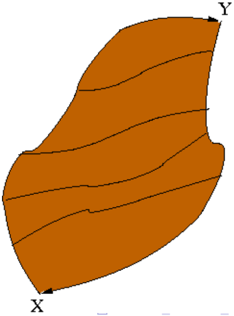
String Representation of Wilson Loop

Assume

$$< W[x(\tau)] >_{YM} = \int DX(\sigma) \exp\{-S[X(\sigma)]\},$$

where $X(\sigma)$ is a surface whose boundary is the given curve $x(\tau)$. Wilson (1974), Nambu(1979), Polyakov(1987)

Then $S[X(\sigma)]$ can be interpreted as the action for an open string whose endpoints are a scalar "quark" and an "antiquark"



Nambu-Goto Strings

Effective Strings

Nambu-Goto action

The simplest action for an open-string is the

$$S_{NG}[X(\sigma)] = T_0 \int d^2\sigma \sqrt{g},$$

T_0 is the string tension, g is the determinant of the induced metric,

$$g_{ab}[\sigma] = \frac{\partial X}{\partial \sigma_a} \cdot \frac{\partial X}{\partial \sigma_b}.$$

An effective description in the spirit of chiral Lagrangian. Polchinski and Strominger (1991), Drummond (2004), Hari Dass and Matlock (2006)

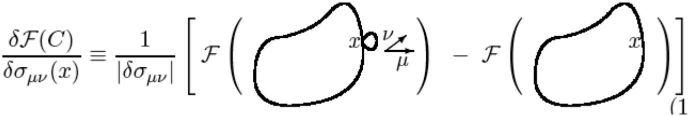
Area Derivative: Wilson Loop to Super Wilson Loop

Area Derivative

The super Wilson loop is not an independent loop functional but is related via a linear operator to the Wilson loop

$$\exp\left\{-\frac{1}{2}\oint d\tau\psi_\mu\psi_\nu\frac{\delta}{\delta\sigma_{\mu\nu}}\right\}W[x(\tau)] = \mathcal{W}[x(\tau),\psi(\tau)],$$

where $\frac{\delta}{\delta\sigma_{\mu\nu}}$ is the area derivative of the loop .



(*Reviewed in Makeenko 1999*)

String Representation of SWL

Expectation value of a SWL

One can write the expectation value of the SWL in terms of the expectation value of the WL,

$$
\begin{aligned}
< \mathcal{W}[x(\tau), \psi(\tau)] >_{YM} &= < \exp\left\{-\frac{1}{2}\oint d\tau \psi_\mu \psi_\nu \frac{\delta}{\delta\sigma_{\mu\nu}}\right\} W >, \\
&= \exp\left\{-\frac{1}{2}\oint d\tau \psi_\mu \psi_\nu \frac{\delta}{\delta\sigma_{\mu\nu}}\right\} < W >_{YM}, \\
&= \exp\left\{-\frac{1}{2}\oint d\tau \psi_\mu \psi_\nu \frac{\delta}{\delta\sigma_{\mu\nu}}\right\} \int DX \exp\left\{-S[X]\right\}, \\
&= \int DX \exp\left\{-S_{SWL}[X, x(\tau), \psi(\tau)]\right\}.
\end{aligned}
$$

Spin-String Interaction for Nambu-Goto String

Nambu-Goto Strings Ending on Quarks

Assuming that the expectation value of a WL can be represented by a Nambu-Goto string and using (Migdal 1983)

$$\frac{\delta}{\delta\sigma_{\mu\nu}}\int d^2\sigma'\sqrt{g} = t_{\mu\nu}[\sigma],$$

where

$$t_{\mu\nu}(\sigma) = \frac{\epsilon^{ab}\partial_a X_\mu \partial_b X_\nu}{\sqrt{g}} = \frac{X_{\mu\nu}(\sigma)}{\sqrt{g}}.$$

implies that the expectation value of a SWL is given by the action,

$$S_{SWL} = T_0\int d^2\sigma\sqrt{g} - \frac{T_0}{2}\oint d\tau\psi_\mu(\tau)\psi_\nu(\tau)t_{\mu\nu}(\tau).$$

String-Spin Interaction and Worldline SUSY

The action for Nambu-Goto string ending on the worldline of a quark

$$S_{SWL} = T_0 \int d^2\sigma \sqrt{g} - \frac{T_0}{2} \oint d\tau \psi_\mu(\tau)\psi_\nu(\tau) t_{\mu\nu}(\tau),$$

can be shown to be invariant under the worldline SUSY

$$\delta x = \epsilon\psi;\ \delta\psi = -\epsilon\dot{x},$$

using the fact that the area-derivatives satisfy a Bianchi-Identity

$$\partial_\lambda(\tau)\frac{\delta}{\delta\sigma_{\mu\nu}(\tau)} + \partial_\mu(\tau)\frac{\delta}{\delta\sigma_{\nu\lambda}(\tau)} + \partial_\nu(\tau)\frac{\delta}{\delta\sigma_{\lambda\mu}} = 0,$$

VV(arXiv:0704.2707 [hep-th])

SWL and Heavy Quark Potential

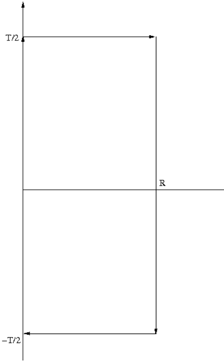
In the non-relativistic limit (reviewed by Peskin 1983)

$$\mathcal{W}_{NR}[T,R] = \mathrm{Tr}\hat{P}\left\{ i\oint dt(x\cdot A) + \frac{1}{4m}\oint dt(\Sigma_{\mu\nu}F_{\mu\nu})\right\}.$$

The expectation value of a rectangular SWL
In the limit $T \to \infty$, can be expressed as

$$< \mathcal{W}[T,R] >_{YM} = \exp\{i\phi(T,R)\}\exp\{-V(R)T\},$$

where $\phi(T,R)$ is a phase factor, $V(R)$ is the spin-dependent potential between the quark and the antiquark separated by a distance R .



String Representation of Rectangular SWL

According to our assumptions the expectation value of a rectangular SWL is given by

$$< \mathcal{W}_{NR} >_{YM} = \int D\phi \exp\{-S_{SWL}\},$$

where

$$\phi(\sigma_1, \sigma_2) = \phi(t, r)$$

are transverse fluctuations around the minimal surface, and the action is

$$S_{SWL}[T, R] = T_0 \int d^2\sigma \sqrt{g} + i \frac{T_0}{4m} \oint \Sigma_{\mu\nu} t_{\mu\nu}(x_0) dt.$$

SWL and the Static Potential

The spin-string interaction can be written as:

$$\begin{aligned} \Sigma_{\mu\nu} t_{\mu\nu} &= \sigma \cdot \mathcal{B} - \sigma \cdot \mathcal{E}, \\ \mathcal{B}_i &= \frac{1}{2} \epsilon_{ijk} t_{jk}, \\ \mathcal{E}_i &= t_{oi}. \end{aligned}$$

The part of the string action for a rectangular SWL that contributes to the static potential

$$S_{SWL}[T,R] = T_0 \int d^2\sigma \sqrt{g} + i\frac{T_0}{4m} \int dt^+ \sigma^+ \cdot \mathcal{B}^+ - i\frac{T_0}{4m} \int dt^- \sigma^- \cdot \mathcal{B}^-$$

where the superscripts $\pm$ denote the quark and the antiquark worldline.

Spin-Spin term in the Heavy Quark Potential

The appropriate boundary conditions for a rectangular SWL are

$$\dot{\phi}(t,0) = \dot{\phi}(t,R) = 0$$

and using

$$t_{ij} = \frac{1}{\sqrt{g}}(\dot{\phi}_i \phi'_j - \phi'_i \dot{\phi}_j)$$

we find

$$\sigma \cdot \mathcal{B}^{+} = \sigma \cdot \mathcal{B}^{-} = 0$$

therefore there is no spin-spin dependent term in the static potential.

Flux Tube and Spin-Spin Interaction

Spin-Spin Interation in a Flux Tube Model of Confinement

According to Kogut and Parisi (1981)

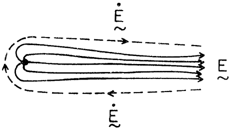


FIG. 1. The fields $\vec{E}$ and $\dot{\vec{E}}$ in the vicinity of the fluctuating string. The string is rotating upward in the plane of the figure about the heavy quark.

$$\mathcal{B}_i \sim \epsilon_{ij} \frac{\partial \dot{\phi}_j}{\partial r}$$

which leads to a spin-spin term in the static potential.

Spin-String Interaction for Thick Strings

Assume

The effect of finite thickness is to smear the spin-string interaction from the boundary. Therefore average the spin-string interaction over a distance of the order of the transverse thickness of the string

$$\begin{aligned}\bar{t}_{ij}(t,x^+) &= \frac{1}{r_T}\int_0^{r_T} dr t_{ij}(t,r) \\ &= \frac{1}{r_T}\int_0^{r_T} dr\left(t_{ij}(t,x^+) + r\partial_r t_{ij}(t,x^+)\right) \\ &= \frac{r_T}{2}(\partial_r t_{ij})_{x^+}.\end{aligned}$$

Spin-Spin Interaction in Thick String

Thick String

Evaluating $\bar{t}_{ij}$ for small transverse fluctuations, $\phi_i \ll 1$,

$$\sigma \cdot \bar{\mathcal{B}}(x^{\pm}) = \frac{1}{M_g}\sigma \cdot (0, \partial_r \dot{\phi}_z(x^{\pm}), -\partial_r \dot{\phi}_y(x^{\pm})),$$

where $M_g^{-1} = r_T/2$ is some measure of the transverse thickness of the flux tube. Apart from the factor of M_g^{-1}, this is precisely the interaction assumed by Kogut and Parisi.

Spin-Spin term in the Static Potential

Spin-Spin Interaction

The effect of the spin-string interaction can be evaluated in perturbation theory

$$V_{ss} = \frac{T_0^2}{(mM_g)^4} \frac{\sigma^+ \cdot \hat{R}\, \sigma^- \cdot \hat{R}}{R^5},$$

where $\hat{R}$ is a unit vector pointing from the quark to the antiquark.

In the limit $M_g^{-1} \rightarrow 0$, which corresponds to a string with no transverse thickness, V_{ss} vanishes and there is no spin-spin term in the static potential.

Open-Questions

- Summing over boundaries = open string with free boundary condition?
- Spin-string interaction for open string with free boundary condition?
- Does worldline SUSY for the effective string implies a worldsheet SUSY for the fundamental QCD string?
- Relationship between fluxtubes and fundamental strings?

Conclusions

- An effective string action for a string ending on the worldline of a quark and an antiquark which has spin-string interaction.
- The resulting action has a worldline SUSY.
- Implies that for a fundamental string there is no spin-spin term in the Heavy Quark Potential.
- If we assume that the effect of finite thickness is to "smear" the spin-string interaction than there is a spin-spin term in the Heavy Quark Potential which goes as R^{-5}.